Model-based Stochastic Control of Traffic Networks

S.P. Hoogendoorn, V.L. Knoop, and H.J. van Zuylen

Paper submitted for the INSTR 2007 Conference, Delft, The Netherlands

Abstract
Uncertainty of traffic network operations has been a subject of lively debate in the last decade. However, little efforts have been put in developing control frameworks that are not only aimed at improving the mean performance of the system, but also at improving the system robustness and reliability. In fact, it can be argued that most of the current control approaches are only aimed at improving the efficiency, which can even be counterproductive from a robustness point of view.

This paper introduces a new concept of computing traffic controls which explicitly takes into account the uncertainty in predicted traffic conditions. This is achieved by including these uncertainties explicitly in the control objective function via the predicted system performance variability. The overall approach is based on the concept of Model Predictive Control (MPC), using the current observed or estimated state of the system as the initial conditions for the uncertain predictions.

The paper makes clear how different performance function specifications yield different control strategies. This is shown for a relatively simple case study.

Keywords
Stochastic control, reliability, robust networks

1 INTRODUCTION

Dutch transportation policy is not only aimed at achieving the largest throughput, but also at increasing the travel times reliability. In fact, current policy aims to achieve that 95% of all motorway trips in the peak hour will be on time. From the perspective of the direct users of the transportation users, this will mean a considerable improvement, because behavioral studies have already shown that besides mean travel time, the travel time reliability plays an considerable role in the valuation of a trip (Bogers et al,2006).

Despite of this, for the deployment of most of the Dynamic Traffic Management (DTM) measures, we generally only take consider efficiency impacts (in terms of maximizing throughput, reducing emissions and noise, etc.). Little research has been done how DTM can be put to use to increase reliability. Amongst the few examples is the work of Liu (2006), showing how tolling can be used to improve reliability.

This paper puts forward a new control methodology showing how to 'control for reliability'. The approach is based on the concept of Model Predictive Control (MPC) including a rolling horizon approach. However, rather than using a deterministic prediction model (which is done in traditional MPC), we now use a stochastic model instead. As a result, the predicted performance is a random variable, rather than a single deterministic value.

2 DESCRIPTION OF THE TRAFFIC CONTROL PROBLEM

Dynamic Traffic Management offers many possibilities to influence traffic flow operations in networks. Examples are providing route information or guidance, ramp-metering, mainline metering, tidal flow, dynamics speed limits, intersection control, etc.

In this paper, we assume that the *status* or *control settings* of these measures can be represented by some control input $\mathbf{u}$. This vector can include all kinds of control settings, such as the green-time, whether or not a specific lane is closed, the route advice people receive via the VMS, etc. Furthermore, the control will be dynamic, i.e. $\mathbf{u} = \mathbf{u}(t)$.

The control $\mathbf{u}(t)$ influences the (current and future) state $\mathbf{x}(t)$ of the system (this is expressed mathematically in the next section). We generally assume that the next state (say, $\mathbf{x}(t+dt)$) is determined by the current state $\mathbf{x}(t)$, the control $\mathbf{u}(t)$, and any 'disturbances' that may be applied (including the boundary conditions, such as traffic flowing into the considered network). This implies that the state captures the entire history of the system.

In the remainder of the paper, we set out to determine the optimal control settings $\mathbf{u}^*$, i.e. the control that steers the state of the traffic network in some optimal way, according to some performance criteria chosen.

3 MATHEMATICAL FORMULATION OF THE CONTROL PROBLEM

In this section, we will formally describe the stochastic traffic control problem. First of all, we will consider the stochastic describing of the system dynamics. This is achieved by writing the system in the so-called *continuous stochastic state-space form*, i.e.:

$$\begin{aligned} d\mathbf{x} &= \mathbf{f}(t,\mathbf{x},\mathbf{u})dt + \sigma(t,\mathbf{x},\mathbf{u})d\boldsymbol{\omega} \\ \mathbf{x}(t_0) &= \mathbf{x}_0 \end{aligned} \tag{1.1}$$

In Eq. (1.1), $\mathbf{x} = \mathbf{x}(t)$ denotes the state vector of the system at time instant t, $\mathbf{u} = \mathbf{u}(t)$ denote the control vector, and $\boldsymbol{\omega} = \boldsymbol{\omega}(t)$ denotes the white noise term. The term $\boldsymbol{\sigma}$ denotes the error variance term. Note that the covariance matrix is defined as follows

$$\Theta = \sigma\sigma' \tag{1.2}$$

Note that for we assume that the noise is additive. This assumption is no restriction to the noise as any noise function can be written as additional function of t, $\mathbf{x}$ and $\mathbf{u}$.

Employee 2-1-07 13:54
Deleted: the sake of simplicity,

Employee 2-1-07 13:54
Deleted: can be relaxed.

3.1 Concept of stochastic costs

Assume that at some time $t = t_k$, the state $\mathbf{x}(t_k)$ of the system is known (at least to a certain extent). From that time instant onward, we can compute the distribution of $\mathbf{x}(t)$ for $t > t_k$, and sample instances from this distribution using Eq. (1.1).
Clearly, since $\{t,\mathbf{x}(t)\}$ for $t > t_k$ is a random process, so is the cost J defined by:

$$J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = \int_{t_k}^{t_k+H} L(s, \mathbf{x}(s), \mathbf{u}(s))ds + \phi(t_k + H, \mathbf{x}(t_k + H)) \quad (1.3)$$

where L denotes the so-called *running cost*, while ϕ denotes the terminal cost; H denotes the prediction horizon.

The cost J defined by Eq. (1.3) is a random variable describing the performance of the system, for instance in terms of cumulative travel times or delays, throughput, average speeds, etc., given that from time t_k to time t_k+H we have applied a certain control $\mathbf{u}$.

Let $E(X)$ denote the expected value of the random variable X. For stochastic control problems, the *objective function* J is generally defined by the expected cost, conditional on the control $\mathbf{u}$ that is applied:

$$\overline{J}(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = E\left[J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)})\right] \quad (1.4)$$

Besides the expected cost mostly used in stochastic control theory, we can for instance also define the conditional cost variability:

$$\Sigma(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = \text{var}\left[J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)})\right] \quad (1.5)$$

Clearly, any other statistic (median, mode, skewness, kurtosis, 85% percentile, etc.) *or combinations of statistics* can be used to express the future performance of the system, given the current state $\mathbf{x}(t_k)$ and the control $\mathbf{u}(s)$ for $s > t_k$. In the remainder, we will use the symbol ϑ to express the chosen statistic describing the system performance:

$$\vartheta(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) = \left\langle J(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) \right\rangle \quad (1.6)$$

where $\left\langle X \right\rangle$ denotes the chosen statistic expressing the future system performance.

3.2 Formulation of the stochastic control problem

The resulting control problem is similar to the deterministic control problem: given the currently available state $\hat{\mathbf{x}}(t_k)$, the aim is to find the control $\mathbf{u}$ optimizing the chosen system performance, i.e.:

$$\mathbf{u}^*_{[t_k, t_k+H)} = \arg\min \vartheta(t_k, \mathbf{x}(t_k), \mathbf{u}_{[t_k, t_k+H)}) \quad (1.7)$$

subject to:

$$\begin{aligned} d\mathbf{x} &= \mathbf{f}(t, \mathbf{x}, \mathbf{u})dt + \sigma(t, \mathbf{x}, \mathbf{u})d \\ \mathbf{x}(t_k) &= \hat{\mathbf{x}}(t_k) \end{aligned} \quad (1.8)$$

It is important to note that the problem has been formulated as a *rolling horizon problem*, where it is assumed that the optimal control is recomputed each time a new measurement or state estimate becomes available. As such, the system can be made responsive to unpredicted changes in the system, such as incidents, bad weather conditions, etc.

3.3 Solutions to the control problem

The theory on stochastic control problems has been a focus of attention since the 90's. A good introduction is given by Fleming and Soner (1993), focusing in particular on optimizing Eq. (1.4) in the case of uncertain system dynamics. It is beyond the scope of this paper to give a complete overview of all available work. Maar hoe is het hier opgelost? Rather, we will show how the choice of different performance specifications will affect the optimal controls.

4 CASE STUDY

We will illustrate the concepts developed in this paper by a simple but not trivial, hypothetical example. Figure 1 shows the network considered in the case study, and the origin-destination relation that is considered. Note that for the sake of illustration, we will only consider a single origin-destination relation and the control thereof. Generalization to multiple origin-destination pairs is conceptually straightforward.

Victor Knoop 14-10-10 16:47
Deleted: Figure 1

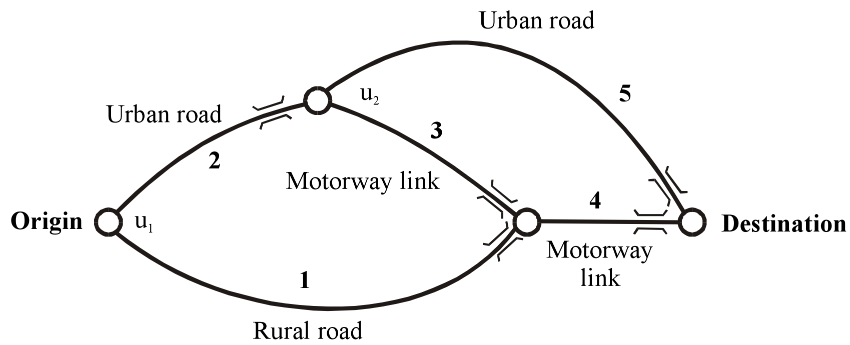

Figure 1 Hypothetical network with controls u_1 and u_2.

In the case, we consider only route guidance control. However, the methodology can be applied to any other kind of control (ramp-metering, main-line metering, intersection control, etc.). The traffic in the network is guided over the links of the network according to the controls u_1 and u_2 respectively denoting the share of traffic going into link 1, and the fraction of traffic flowing into link 3.

Note that different link types are present in the example. We will assume that all links have a different (stochastic) capacity and travel time, amongst other things depending on the type of link. With respect to the capacity, two kinds of uncertainty will be considered. The ‘regular’ uncertainty due to natural variations in traffic composition, driver behavior, etc., and the ‘irregular’ uncertainty due to incidents, accidents, changing weather conditions, etc. Besides a random capacity, we can also assume that the traffic demand is stochastic. It is important to note here that the nature of the variations in the demand is very different from that of the supply. Statistical analyses have shown the importance of including autocorrelation terms in the modeling of demand variations.

4.1 Stochastic model of traffic demand and traffic operations

Since this paper only deals with the general concept of stochastic or robust traffic control, we will use a rather simple model to describe network traffic conditions. It is however emphasized that in principle, any stochastic model (either analytical or simulation) can be used. More specifically, a *vertical queuing model* with a random capacity is used (also known as the QUAST model; see (Stembord,1991)). For the sake of simplicity, the randomness in the capacity is for the case study only determined by the probability of an incident occurring. When an incident occurs, it is assumed that the link is fully blocked for a random period of time (uniformly distributed). Note that the incident probability is determined by the link type (rural link: 10%, urban link: 25% and motorway link 1% - erbij zetten in figuurtje?)[1]. Spillback is not modeled in this simple example, since it is not necessary to illustrate to concept.

For a first traffic demand and for a fixed control law **u**, the dynamic stochastic model is ran N times, leading to N different performances J, using a fixed random seed for realization i (meaning that realization i will always yield the same incident characteristics). Based on these N realized performances, the different relevant statistics can be computed (e.g. mean performance, performance standard deviation, 85% percentile, etc.).

4.2 Case study results

Table 1 shows an overview of the performance of the network. The *collective travel times* have been used as the basic performance indicator. The table thus shows the mean collective travel times, the 95% percentile of the collective travel times, the median thereof, and the standard deviation. At the same time, Figures 2-4 show the optimal controls u_1 and u_2 applied during the period.

Victor Knoop 14-10-10 16:47
Deleted: Table 1

Table 1 Network performance for different control objectives (objective function).

Objective function	mean(J)	median(J)	$F^{-1}(0.95)$	std(J)
mean(J)	**3479**	3398	3828	279
median(J)	3493	**3379**	4123	320
$F^{-1}(0.95)$	3526	3460	**3665**	272

Figure 2 shows the same results graphically.

Victor Knoop 14-10-10 16:47
Deleted: Figure 2

[1] The incident probabilities have been chosen large to more clearly illustrate the approach.

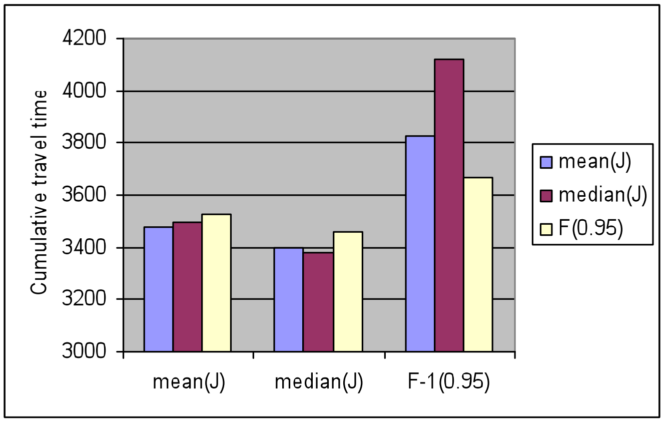

Figure 2 Cumulative travel times per control strategy (optimizing mean performance, median performance and 95% performance).

Victor Knoop 14-10-10 16:47
Deleted: 2

The differences between the results per objective function are clear. For instance, if we compare what happens when optimizing the median collective travel time compared to the mean collective travel time, we see that on average u_1 decreases. This means that a larger part of the traffic is diverted via link 2 (motorway route). Next, we see that a relatively small part chooses link 3 compared to the situation where the mean is optimized, implying that less traffic is guided along the reliable routes. This is as expected, since the median is less sensitive to outliers than the median.

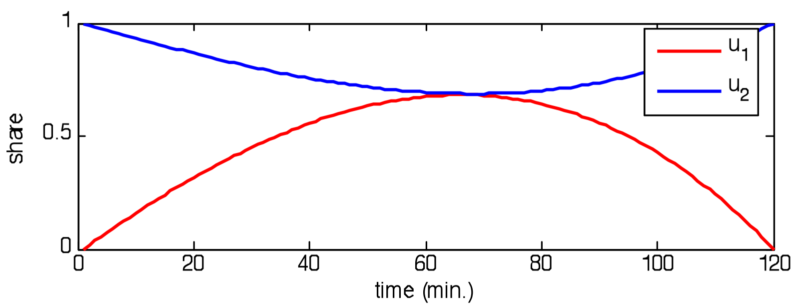

Figure 3 Dynamics of the optimal controls u_1 and u_2 for the two hour simulation period when the mean travel time is optimized.

Victor Knoop 14-10-10 16:47
Deleted: 3

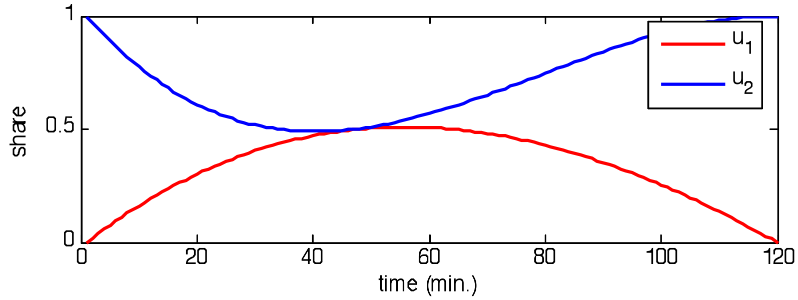

Figure 4 Dynamics of the optimal controls u_1 and u_2 for the two hour simulation period when the median travel time is optimized.

Victor Knoop 14-10-10 16:47
Deleted: 4

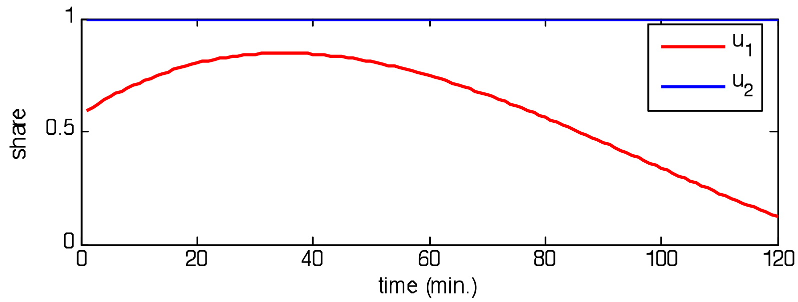

Figure 5 Dynamics of the optimal controls u_1 and u_2 for the two hour simulation period when the 95% percentile travel time is optimized.

Victor Knoop 14-10-10 16:47
Deleted: 5

5 CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed a new optimal control paradigm for traffic networks explicitly including the uncertainty of the resulting traffic operations. The approach was illustrated by means of a simple application example. The example shows clearly the impact of the control objective function on the resulting optimal control strategies.

REFERENCES

1. Fleming, W.H., and H.M. Soner (1993). *Controlled Markov Processes and Viscosity Solutions*. Springer-Verlag.
2. H. Stembord (1991). Quality of service on the main road network in the netherlands. In B. Brannolte, editor, "Highway Capacity and Level of Service", Rotterdam.