Network robustness assessment: the need for spillback modelling

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Abstract

The quality of a road network can be defined as the degree to which it offers reliable connections under all prevailing conditions, an aspect valued by road users. In this light, network robustness is an important study area for transportation scientists now. This paper studies one specific aspect of robustness: the consequences of the blocking of a link in a road network using a traffic simulator. A special feature in the new simulator proposed here is that spillback can be switched on and off; also, the route choice can be updated or not. The paper describes effects of spillback on robustness explicitly. We conclude three things. First, the correlation of the vulnerability of links between simulations with spillback and without spillback is low, so any study to identify vulnerable links should include spillback. Second, network performance drops more if a motorway link is blocked then if another link is blocked. Finally, computations show that route adjustments are more effective if no spillback is simulated.

Keywords

Network robustness, network vulnerability, link blocking, spillback, macroscopic simulation

1 Introduction

The reliability of road networks is valued by both travellers and network authorities (Bogers *et al.*, 2005; Liu *et al.*, 2004). Bates, for example, found that a one minute reduction of standard deviation of travel time and a two minute reduction of actual travel time are equally valued (Bates *et al.*, 2001). Furthermore, (Bogers and Zuylen, 2004) showed that drivers avoid routes that are on the average the quickest but have a probability on exceptionally high travel times. The robustness of a network is the ability of a network to cope with variations in demand or network capacity without much influence on travel times and is as such a corner stone for travel time to reliability. The mentioned variations can be caused by normal daily fluctuations in demand and supply as empirically shown by (Tu *et al.*, 2005) . Another cause for variation is the blocking of a link by an incident or road maintenance. This is not part of the normal daily fluctuations and reduces the capacity of a link significantly.

This paper analyzes this reduction of performance of a network caused by an incident. Different research projects assessing road network robustness use different traffic simulation models. The models differ in the dynamics of traffic flow and congestion. Due to the complexity of the network modelling, sometimes spillback, i.e. congestion propagation to a more upstream link, is not modelled. This contribution therefore compares link network robustness for scenarios with and without spillback in order to assess the need of proper spillback modelling. This study makes a distinction between cases in which road users adapt their route choice to the new situation with a blocked link and congestion and a situation in which they do not. The research investigates too for the blocking of which links the network performance is particularly sensitive.

The method we use is as follows. We sequentially block all links, one at a time, and compare the results in network performance. The robustness is inversely proportional to the number of links that reduce the network performance in a considerable way. The robustness of the network depends on the internal structure and the flows in the network.

The main question in this paper is if conclusions about robustness and properties of important links differ for simulations with spillback and without spillback and if robustness studies using network models without spillback have any practical value. It is found that the robustness assessment differs considerably for the two approaches. Therefore spillback should be well modelled in robustness studies.

The paper is set up as follows. First, it investigates which approaches to robustness are used. We will then formulate the mathematics of one approach. Different simulation studies can be based on this mathematical framework. The section “traffic flow modelling” describes the differences between these simulation studies. We will compare the simulation techniques in a case study that is described in the next section. The last two sections present the results and the conclusions.

2 State of the art

A considerable part of the delays faced by travellers, about 25% at least, is caused by incidents causing link failure (Kwon *et al.*, 2006). In this paper, we discuss these delays in detail.

The robustness against link failure is usually approached in one of the following two ways. The first one, used in (Bell, 2000; Bell and Cassir, 2002), is an analytical, game theoretical approach. One player, an “evil entity” wants to destroy a network and is given the possibility to destroy a limited number of links. The road users, the other players, choose their routes in order to get a short travel time. Usually, it is assumed that drivers take the route such that they can not unilaterally change their route and get a higher utility, usually based on travel time (Wardrop, 1952).

Those links that reduce the network performance most if they are blocked are called vulnerable. The lower the effect of the dropout of links, the more robust a network is considered to be. In this approach, results are based on an analytical approach and a mathematical framework is set; in both articles, (Bell, 2000; Bell and Cassir, 2002), a simple network is used as test case. The same approach is also used in (Murray-Tuite and Mahmassani, 2004), but here, the travel times also depend on the flow on the links.

The other approach found in literature, is the identification of vulnerable links in a real-size network, for instance using simulation. In (Knoop *et al.*, 2005), properties of vulnerable links are presented based on a simulation study. A method to find vulnerable links quickly in large networks is proposed in (Tamminga *et al.*, 2005), but in an aim to reduce computation time, the traffic dynamics are not captured correctly. In (Tampère *et al.*, 2006) a method is presented to select the most vulnerable links in different stages. The first stage selects potentially vulnerable links and links that do not need to be reconsidered for being very vulnerable. This first selection is based on link flow properties in the every day situation.

The traffic representation in all of these models differ; one of the important differences is whether spillback is included or not. This difference is investigated further here. Our research aims at problems in a real-world network. We will perform a simulation study in which we will incorporate the mathematical concepts of the analytical studies.

3 Mathematical approach

There are two (groups of) actors in the situation at hand: on the one hand the travellers all wanting the best path through the network and on the other hand an “evil entity” wanting to harm the performance most by blocking one link. The network robustness studied here can be easily related to the performance of the evil entity. If a link is blocked, the evil entity wants the network performance to reduce a lot. Road users, on the other hand, all aim for a short travel time independent of the link blocking. The collective result of the trips of all travellers is a measure for the network performance. As road users want to optimize their individual trip under all circumstances, the aim for the collective of road users is a robust network, although in general the individual optimal route choice does not lead to a collective optimum.

The situation is studied in two ways here. The first way is that the road users choose the initially fastest routes and do not deviate from it, even if their route is congested or blocked. In the second way, a road is blocked and road users will adapt their routes according to the new situation; this will be the case if drivers are informed about the real-time traffic situation.

3.1 Static, predetermined routes

In this scenario, routes are fixed at the beginning of the simulation according to the Wardrop principle: no one can unilaterally change his route and get a shorter perceived travel time to his destination; these routes are referred to as π_0^* (G), in which G is the network these routes are based on. For reasons of simplicity, routes are

in this case based on an undisturbed network and free flow travel times which is indicated with the subscript *0*. In practice, and also in our simulation, routes are subject to personal valuation and perception, even if they are based on free flow conditions. Therefore, in the simulation program, a number of sets of link travel time perceptions are drawn. The link travel time is drawn from a normal distribution with the calculated travel time as mean and a standard deviation of 10%. Each set of link travel time drawings determines a set of routes from each point to each destination. A large number of drawings can be considered representative for the whole population and so routes are assigned via a probit procedure.
For the trips from every node in the network to each destination a route choice is now made. Then, one link is blocked and the network G changes into G_b. For every link blocked, a separate simulation is run.
For the evaluation of the performance of a network an indicator should be used. We want this performance indicator to be lower if people need to take a detour, come later or do not arrive at their destination at all. We opted for the total number of travellers that arrive at their destination in the simulation time as performance indicator, A. Another choice in line with the user’s wishes is the average travel time. However, this requires extra care in interpretation and calculation. It needs for example to be decided for which number of travellers the average is calculated and comparisons would be more complicated. We therefore choose for the number of people arriving at their destination as performance indicator. We do not have an indication that the results would be different if we took for instance average travel time as criterion.
The network flows and therefore also the network performance A can be different for the scenario in which spillback is modelled and the scenario in which no spillback is modelled, so the performance A also depends on the simulation of spillback, ss.
The performance of the network in this scenario can now mathematically be expressed as:

$$A\left(\pi_0 * (G), G_b, ss\right). \tag{1}$$

The most vulnerable link $b*$ is the link for which the network performance is lowest if it is blocked:

$$b* = \underset{b}{\operatorname{argmin}}\left(A\left(\pi_0 * (G), G_b, ss\right)\right). \tag{2}$$

3.2 Dynamically adapted routes

Here, the simulation starts with blocking link b. The travellers respond to that action by changing their routes and therefore, their actions are different for every different link blocked.
The road users adapt their behaviour to the newly envisaged situation in the blocked network, and will also change their routes during the simulated time due to congestion. In this case, the routes are based on the actual traffic situation with a blocked link. People will choose the route that is in their view the fastest at the moment of choice, at the moment, $\pi*(G_b)$. No iteration is applied to account for the congestion in the coming time period. As we assume that people base their routes on the actual traffic information, user equilibrium conditions are not likely to occur in a non-recurrent situation with an incident. The assignment is, similar to the other scenario, done by a probit assignment. The routes are now updated every time period of fifteen minutes due to congestion. In practice, this is for example possible by regular in-car information of the speeds.

Link travel time is computed from link distance and average speed on the link, which, in turn, is the calculated flow divided by density. Travellers choose the route to their destination that takes the shortest time in their view. Also in this scenario, link travel time perceptions are drawn from normal distributions with the link travel times as mean and a 10% standard deviation. Based on the set of link travel times, the perceived best routes and therefore the route choice can be determined. This is repeated a number of times to obtain a set of route choices, according to which all travellers get a route assigned. At the end of a period, this procedure is repeated with updated speeds. Due to the heterogeneity of the drivers, half of the travellers will be assigned to a new route; the other half will choose the old route for the coming period. If the fraction of people that take a different route is too small, the route choice effect cannot be seen. On the other hand, a too large compliance rate is unrealistic (Bogers, Viti, 2005).
As network flows will differ for scenarios with and without spillback, the network performance can also differ dependent on the simulation of spillback.
For this scenario, the network performance function that needs to be evaluated is

$$A\left(\pi^*\left(G_b\right), G_b, ss\right). \tag{3}$$

This function is calculated for each link b, so the most vulnerable link b* is:

$$b^* = \underset{b}{\operatorname{argmin}}\left(A\left(\pi^*\left(G_b\right), G_b, ss\right)\right). \tag{4}$$

In terms of a mathematical game, this link b^* would be the Stackelberg optimum for an evil entity to block if he was given the opportunity to block one link.

4 Traffic flow modelling

Analysis to road network robustness can be done with two types of simulations. Both microscopic simulations, in which vehicles are individually simulated, and macroscopic simulations are used. In microscopic simulation models, the quality of the simulation depends on the quality of the underlying simulation sub-models. These models describe the driver tasks, such as car-following models or lane change models. In theory, spillback can be captured correctly as all cars occupy a finite space.
Simulation studies that cover a larger area often use macroscopic models to reduce computation time. With these models, it is an issue to implement the congestion flow crossing a link border in a correct way. Often, vertical queuing models are used to describe queues. In vertical queuing models, traffic jams do not occupy any space. What matters for the traffic flow, is that flow at the upstream link is not influenced by the queues on the (downstream) link. In our representation of a non-spillback model, the queue will grow upstream but does not cross the link border. The queue dynamics of this model are plotted in Figure 1a; in the gray shaded area are cars queuing.
In reality, the queue will grow more upstream than the end of a link. This can be captured by a horizontal queuing model. In this model, the head of the queue is fixed at the bottleneck and the queue can only dissolve from the tail. Often, a fixed space per car is assumed. The queuing dynamics of a horizontal queuing model are indicated in Figure 1b. In the lower figure, we assumed that every car in the upstream link would also turn on the congested link. In that case, no more cars are jammed. However, if the link crossing is a junction with traffic for other directions, the queue might grow faster at the end.

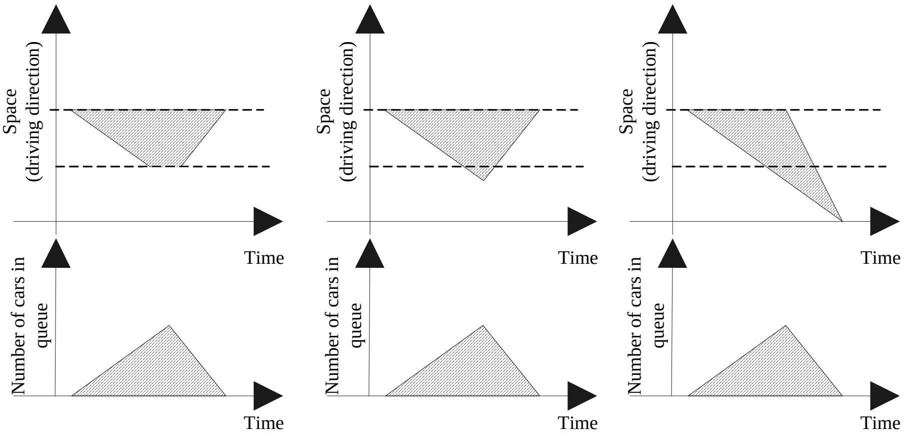

Figure 1: Congestion dynamics in a space-time plane (upper figures) and number of cars in a queue (lower figure). From left to right: implementation of a) a vertical queuing model, b) horizontal queuing model, c) shockwave theory

The queue will, possibly, dependent on traffic conditions, dissolve from the head, while the tail of the queue still moves upstream, as seen in Figure 1c. This study compares the situation with vertical queues (as in Figure 1a) with the situation of full spillback and tail solving from the head of the queue (Figure 1c). For the number of cars in the queue, we plotted the case that every vehicle wants to enter the congested link, and the number of cars in the queue is therefore the same. In another assumption, the queue could grow further upstream involving more cars.
Shockwave theory is needed to describe the real traffic state (Figure 1c) behaviour sufficiently accurate. We use a continuum LWR-model that we solve with a Godunov scheme (Lebacque, 1996). Second order effects (e.g. from synchronized flow to wide moving jams) have some minor influences (Ngoduy, 2006) that we will neglect in the remainder of this article.

5 Case study description

The best way to compare the results of the vulnerability of links in a spillback and in a non-spillback simulator is to use one simulator that can run simulations both with and without spillback. In this way, there are no differences in systematic errors.
We used the simulator DSMART developed at Delft University of Technology. The working of the simulator is discussed briefly here; for more details, we refer to (Zuurbier *et al.*, 2006). The macroscopic simulator splits links in cells. In each cell, the density and the average speed are assumed homogeneous; these two variables determine the state of the cell. A fundamental diagram gives a relation between density and flow; we used a triangular shaped fundamental diagram (Daganzo, 1997). This fundamental diagram is used in a Godunov scheme (Daganzo, 1995): according to the density and this fundamental diagram, every cell has a certain output demand, increasing with density, and a supply ability, decreasing with density. The minimum of the demand of the upstream cell and the supply of the downstream cell is the flow from one cell to the adjacent cell:

$$Q_{i,i+1} = \min\{\text{demand}_i, \text{supply}_{i+1}\}. \tag{5}$$

If at junctions the combined demand of the upstream links to one downstream link is larger then the supply of the downstream link, the transmission flow from all upstream links is reduced with an equal factor. In this way, vehicles queue at the

upstream link and so congestion propagates upstream and spillback is put in the model.
A modification of the model excludes spillback from the model. In this changed model, the supply of the most upstream cell of a link was always, independent of the congestion state, capacity. In this model, a traffic jam on the downstream link does not influence the traffic state on the upstream link.
As a case study, we use the network of freeways and the main roads of the underlying road network around the Dutch city of Rotterdam (around 600.000 inhabitants, see also Figure 3a. The network has around the same size as the maximum influence of an incident: no effects will occur much further away. Therefore, it is not necessary to consider a bigger network.
468 links are modelled. The network model and the dynamic OD matrices are calibrated qualitatively on a morning peak period. Our simulation covered a morning peak from 6.30 to 9.30 am. The total simulation time for 468 (blocked links) times 4 (scenarios) runs is approximately two weeks on a 2.8 GHz PC.
The incidents are modelled as follows. During the whole simulation, the capacity of a link is reduced. The new capacity of the blocked road is expressed mathematically as:

$$C_{blocked} = C_{old} \max\left\{ \frac{nrlanes - 2}{nrlanes}, 0 \right\} \tag{6}$$

in which C stands for the capacity of the links. This means that at most two lanes will be blocked.

6 Results

Four different scenarios are compared: 1) no spillback, static route choice; 2) no spillback, dynamic route choice; 3) spillback, static route choice; 4) spillback, dynamic route choice. In this section, we compare the results of these simulations, starting with some general findings. Then we analyze the advantage of path updates in cases with and without spillback. In the next subsection, we compare the network robustness in cases with and without spillback. In the last two subsections, we assess the impact of the blockings of the individual links. First, we see that the blocking of a freeway link has the most impact on the network performance. In the last subsection, it is set out why a network performance reduction computed without spillback does not give information on the network performance reduction with spillback simulated.

6.1 General findings

Comparing the results of two situations only differing in spillback (so the same link blocked, same settings for route choice), in the simulation without spillback, more travellers arrive at their destination in the simulation time. At the end of this section, we will discuss this result, more extensively. Comparing the results of two situations only differing in path update (so the same link blocked and the same spillback model used), in the simulation with path update, more people arrive (in almost all cases). Both results are obvious and according to the expectations. More relevant and interesting results can be found in the next subsections.

6.2 Performance increase due to route updates

If the results are analyzed more closely, there appear to be interesting differences between several scenarios. From the results we computed the relative advantage of updating the paths:

$$adv(b,ss) = \frac{A(\pi^*(G_b),G_b,ss) - A(\pi_0^*(G),G_b,ss)}{A(\pi_0^*(G),G_b,ss)}. \tag{7}$$

For both cases of spillback simulation, we get 468 values, one for every blocked link. These values were ordered and plotted, as cumulative distribution, in Figure 2a. It shows that the advantage of updating paths is computed to be much bigger if spillback is not simulated.

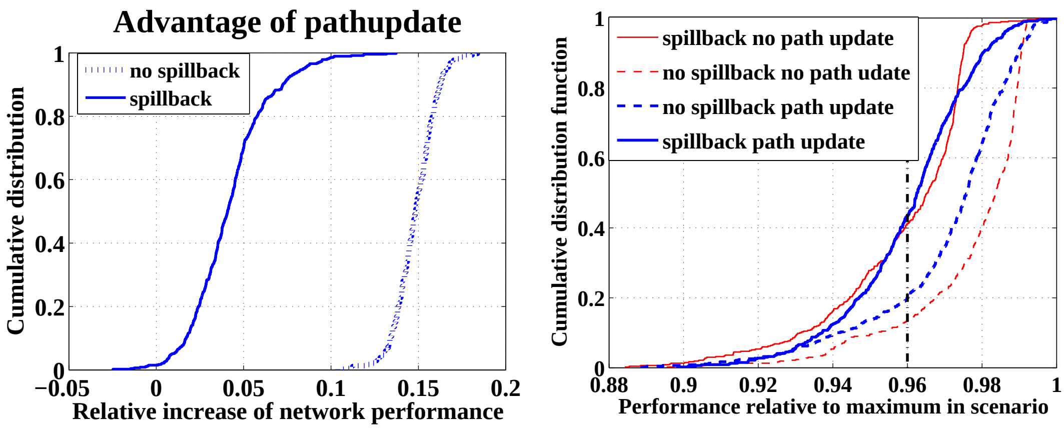

Figure 2: a) Cumulative distribution function of the advantage of the network performance when applying path updates; b) the cumulative chances that the network performance decreases more than a certain level (horizontal axis) if a randomly chosen link is blocked.

6.3 Network robustness with and without spillback simulation

The assessment of the robustness of the network is done by evaluating the impact of blocking single links. In Figure 2b, the cumulative distribution function of the performance indicator relative to the maximum number of each scenario is plotted. Note that this plot is not a distribution function of the *absolute* network performance.

From this figure, we can see what the (relative) performance of the network is if a randomly chosen link is blocked. This can be related to robustness in the following way. We will take a threshold value for the network performance decrease and find the probability that the decrease is more, given that one randomly chosen link is blocked. If that probability is small, the robustness is high.

This threshold value should be a typical value for the performance reduction if link is blocked. This depends for instance on the network size and chosen performance. In the case at hand, a good value is 4% drop of performance compared to the maximum performance obtained in the scenario (generally no link blocked, except a possible Braess's paradox (Braess, 2005)). This is the 0.96 mark at the x-axis. The according values at the y-axis indicate the chance that if a randomly chosen link is blocked, the decrease of the performance compared to the maximum performance is more than 4%. In Table 1 below, these values that can be read from the graph are stated.

Table 1: Percentage of cases for which less than 96% of the maximum number of travellers arrive

	Spillback	No spillback modelled
No path update	41 %	13 %
Path update	42 %	20 %

From this table, we can see that without modelling spillback a big error is made. If congestion spillback is not taken into account, the performance reduction caused by the blocking of one randomly chosen link would in (at most) 20% of the cases exceed 4%. This percentage of vulnerable links will increase sharply if spillback is modelled: it doubles from at most 20% to more than 40%. Keep in mind that the indicated performance drop is the drop *in a certain scenario.* So, the difference in vulnerability is not a consequence of the natural lower performance of a simulation in which spillback is included. For the same reason, it would also be wrong to conclude that once spillback is included, the rerouting does not increase the absolute performance any more. There are indications, though, that the routing will not influence the computed robustness of a network. To really prove this convincingly, more research to the robustness and the actual route choice is needed.

6.4 Blocking of freeway links biggest impact

For each of the links blocked, the network performance can be calculated. The impact from the blocking is dependent on the link. From the data, we see that heavily used freeways are, independent of the scenario, the most vulnerable. If one of these links is blocked, the network performance drops most.
For one particular type of link, the difference in impact for the scenarios with spillback and without spillback is bigger than for other types. These are the connecting links in freeway junctions. If they are blocked in a non-spillback scenario, they turn out not to be that important. However, if one of these connecting links is blocked in a spillback scenario, the network performance reduced considerably more. Figure 3b shows the difference in calculated impact of link blocking. The shown freeway junction is the junction in the oval in Figure 3a. The more the impact factor of the links differ, the darker their colour in links Figure 3b.

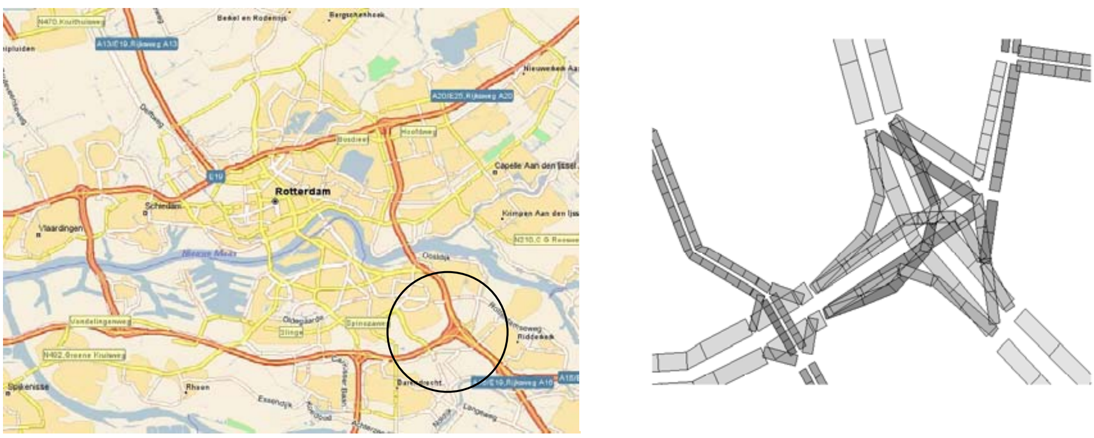

Figure 3: a) map of the study area b) the difference in link impact caused by spillback modelling

6.5 No correlation in robustness values with and without spillback

Finally, we compare the impacts of the blocking of the distinct links on the network performance for both the spillback and non-spillback cases. This is plotted in Figure 4. We carry out a correlation analysis to see how well the computed impact of the blocking of a link can foretell the impact of the blocking of the link if spillback is included.
The x-axis indicates the number of arrived travellers in the case without spillback modelled, the y-axis gives the number of arrived travellers in the case with spillback modelling. A marker in this area indicates the results of two simulations, with and without spillback, in the same settings for route choice (the two different settings are indicated with different markers).

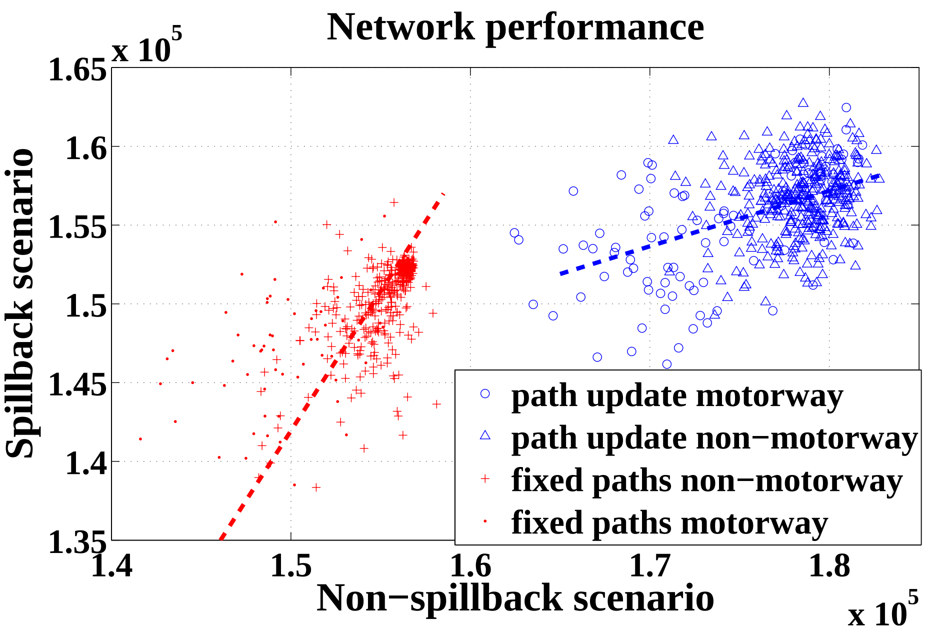

Figure 4: Comparison of network performances with and without spillback

As indicated in the beginning of this section, it is obvious that the network performance without spillback modelled is higher than with the spillback modelled. A possible exception is that a queue has less influence if it is located further upstream due to the position in the network. Almost all results from our case study indicate that the performance in the non-spillback case is higher. Note that the axes are not equal.
Let us now consider both routing cases separately and evaluate the predictive value of the arrivals in the non-spillback case for the arrivals in the case with spillback. To check the correlation between these two values, we performed a linear regression analysis on both (paths fixed and paths update) clouds in Figure 4. Based on the fact that there is not a function describing the exact shape of the cloud Figure 4, we opted for a linear regression. We only consider each of both path update scenarios separately. The data points for the case with path update have a higher value than those without, but that is not the regression we consider here. The two regression lines are also plotted.
The resulting regression coefficients R^2 are 0.20 for the case with path update and 0.39 for the case without a path update. These values mean by definition that only 20% (39% for the case without rerouting) of the variation in arrivals in the case with spillback can be explained by the variation in arrivals in the corresponding scenario without spillback. The conclusion is that simplified simulations that do not take

spillback into account have very limited value for the assessment of network robustness.

7 Conclusions and further research

We simulated a morning traffic flow on a real, regional sized, network for which sequentially one of the links was blocked. The traffic simulator had the possibility to make choices about both path update and spillback. Paths could be adapted to the situation or not and, independently, spillback could be switched on and off.
The first conclusion is that in a network the advantage of rerouting is bigger without spillback modelled than with spillback modelled. This is found for a peak period in which one link is blocked. This advantage can be explained as follows: without spillback modelled, congestion on a limited number of places can be well avoided.
Secondly, we analyzed the decrease of network performance due to a drop out of a random link compared to a fully operational network. There is a difference in the prediction of performance loss. The loss is considerably lower when the estimation is based on a simulation in which spillback is not taken into account, even when computed relative to the performance of the fully operational network. Consequently, robustness is overestimated if spillback is not simulated.
Thirdly, we computed the impact of the blocking of a link on the network performance. Comparison of scenarios with and without spillback showed that there is at most a very weak correlation between this impact if computed with or without spillback. Therefore, any realistic study of the impact of the blocking of a single link must include spillback.
In this research only two route choices were considered. People could either take the route that appeared to be the fastest in free flow conditions or they could adjust the route to the actual traffic situation. In further research, the fixed routes can be made dependent on the daily congestion pattern. Also a third route choice can be added. In this case, road users, aiming for reliability, anticipate on a possible drop out of a link.
In further research also the horizontal queuing model (Figure 1b) can be examined more closely. The difference between this model and the spillback model considered in this research is mainly the location of the queue. When does the location of the queue have a big influence?
The results show that the computed values for network robustness are, once spillback is included, similar for the cases with and without rerouting. It would be interesting to test this conclusion for a wider variety of routing measures and network. In line with this, a whole branch of research could focus on routing measures to increase network reliability.
It would furthermore be very interesting to apply the concepts stated here to an urban network. In short streets with many traffic lights spillback seems an important source of delay. Density waves and shock waves propagate over long distances in the network.

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