Simulation Model for Traffic using Network Fundamental Diagrams

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Abstract Traditionally, traffic is described at the level of an individual vehicle (microscopic) or at the level of a link (macroscopic). This paper introduces a traffic flow simulation model at a higher, subnetwork scale. The network is split in cells (subnetworks), and for each of the cells the Network Fundamental Diagram (NFD) is determined. Each time step, the flow from one cell to antoher is determined by the NFD, separated in a demand and a supply. For the demand, the border capacity between two cells plays a role. Opposed to the cell transmission model, the demand is decreasing for overcritical accumulation in cell i due to accumulation effects. This model can be used to quickly determine effects of network wide traffic control.

1 Introduction

Nowadays, due to increased communication techniques, traffic control measures can be coordinated over larger areas. For this, control concepts need to be developed. Moreover, these concepts needs to be tested, possibly on-line, for which traffic simulation programs are used. We argued earlier [5] that the larger the area, the longer the look-ahead period. For larger areas and longer time intervals, microscopic (vehicle-based) or macroscopic (link-based) simulation programs are too slow.

On an aggregate level there is a relation between the number of vehicles and their speeds [2, 4], the Network Fundamental Diagram (NFD). Although some basic calculations have been describing the dynamics of a network by NFDs, none of these describe a multi-zone network, taking physical effects of the limited bound-

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ary capacity into account. This paper develops such a model. This model is useful for on-line optimization of traffic management measures. The paper also shows the application of the model in section 5.

2 Simulation of traffic flow dynamics

In traffic flow theory, several macroscopic models are available. One of the most intuitively understandable is the cell transmission model (CTM) [1]. In this model, the road is split in cells. The flow between cells is based on an upstream demand and a downstream supply (see also [6]). Up to the critical density, demand is an increasing function. For densities higher than the critical density, the demand is equal to the capacity of the road. The supply equals the capacity of the road up to the critical density. For higher densities, the supply decreases. The flow from one cell to the next is the minimum of the upstream demand and the downstream supply.

The above models describe how traffic flows on links. A network model also needs to describe how traffic behaves at nodes. A good overview of node models and their requirements is given by [8]

In this paper, we combine the concepts of the cell transmission model and good node models and apply it to a model describing the network dynamics, called the Network Transmission Model.

3 Traffic coding

The basis of the model are subnetworks, called cells in the description of the computational methodology. The basic quantities used in this paper are accumulation K and performance P, which can be seen as weighted average density and flow, respectively. Note that performance is the flow which exits a network, rather than the internal flows. It has been shown that the performance is strongly correlated with the internal flow, the production [4]. The accumulation K in each cell A is the average density k for all links Z in the cell weighted to their length L and the number of lanes l. This total weighting factor is indicated by w

$$w_A = \sum_{Z \in A} L_Z l_Z \tag{1}$$

The accumulation is now calculated as

$$K = \frac{\sum_{Z \in A} k_Z L_Z l_Z}{w_A} \tag{2}$$

For each cell, it is registered which fraction of the vehicles (and thus accumulation) is heading towards which destination s; this is called ζ_s. The routing from cell

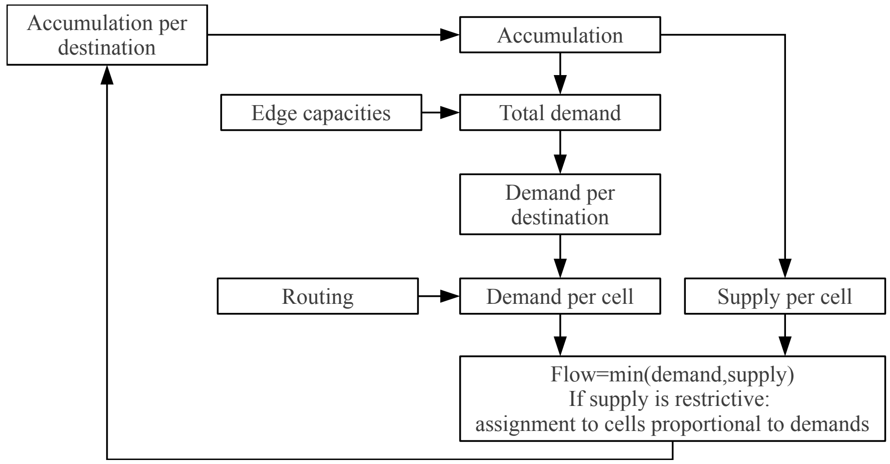


Fig. 1 A graphical representation of the steps taken in the computation scheme

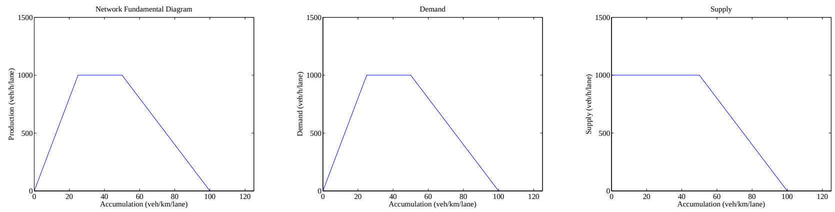
A to the destination is coded by the next neigboring cell B in so called destination-specific splitfractions $\eta_{s,A,B}$. All neighbouring cells of A are indicated by the set $\mathscr{B}$. The fraction $\eta_{s,A,B}$ lies between 0 and 1, and all vehicles should be heading somewhere, so $\sum_{B\in\mathscr{B}} \eta_{s,A,B} = 1$. In our formulation, vehicles are assumed to have arrived their destination once they arrive somewhere in the cell. This could be changed in a future version.

4 Traffic dynamics

Let's now consider the traffic dynamics. The flow diagram of the model can be found in figure 1. The dynamics of traffic are simulated in these subnetwork, using properties of the NFD in each subnetwork. For these cells the NFD is assumed to be known. The flow from cell A to cell B is determined by the minimum of three elements

1. The capacity of the boundary between cell A and cell B, C_A^B; this is determined exogenously
2. The demand from cell A to cell B, D_A^B
3. The supply in cell B, related to the total demand to cell B

The demand from A to B D_A^B is determined based on the NFD, the function which relates production P to the accumulation K: $P = P(K)$. This NFD has to be determined exogenously, for which are several methods, empirically [4] or theoretically [7]. Now, a demand and supply scheme similar to the cell transmission model [1] is constructed. The supply S can be determined in the same way as in the cell transmission model, that is, it is at capacity if the accumulation in the receiving cell is lower than the critical density and equal to NFD for higher accumulations:



(a) Network Fundamental Diagram (b) Demand (c) Supply

Fig. 2 The factors determining the flow

$$S = \begin{cases} P_{\text{crit}} & \text{if } K \leq K_{\text{crit}} \\ P(K) & \text{if } K > K_{\text{crit}} \end{cases} \tag{3}$$

Contrary to the CTM, the demand in a cell *decreases* with an increasing accumulation at values over the critical accumulation. This is because there is internal congestion in the cell, limiting the potential outflow. We thus have: $D = P(K)$, graphically shown in figure 2. Additionally, a minimum flow can be defined. This would allow a demand even from a completely full cell.

The total demand from cell A to cell B, D_A^B is only a part of the total demand in cell A, D_A. In fact, we consider the destinations separately. Hence, the demand in A for each of the destinations is

$$D_{A,s} = \zeta_s D_A \tag{4}$$

For each of these partial demands, the fraction heading to neigbouring cell B is indicated by $\eta_{A,s}^B$ The demand from cell A towards cell B hence is $D_A^B = \sum_{\text{all destinations } s} \eta_{A,s}^B D_{A,s}$. This is now limited to the capacity of the boundary between A and B, C_A^B, giving the effective demand $\widetilde{D}_A^B = \min\left\{D_A^B, C_A^B\right\}$ The fraction of traffic allowed over the boundary between A and B, θ_A^B, is now calculated as: $\theta_A^B = \min\left\{\frac{\widetilde{D}_A^B}{D_A^B}, 1\right\}$ As an intermediate step, we now have the effective demand from cell A to destination s via cell B:

$$\widetilde{D}_{A,s}^B = D_{A,s}\eta_{A,s}^B\theta_A^B \tag{5}$$

The total demand towards cell B (D^B) is determined by adding all effective demands towards cell B, i.e. for all destinations and origin cells A. This is compared with the supply in cell B. If the supply is larger, the flow is unrestricted. However, if the supply is lower, the fraction of the flow which can flow into cell B ψ^B is calculated: $\psi^B = \min\left\{\frac{S^B}{D^B}, 1\right\}$.

All cells B, neighbours of A, which have effective demand $\widetilde{D}_A^B$ larger than zero are combined in set $\mathscr{B}$. It is now calculated what is the lowest of these outflow fractions. This will be the restricting factor for the flow from cell A: Ψ_A: $\Psi_A = \min_{B\in\mathscr{B}}\left\{\Psi^B\right\}$.

If the supply restricts the flow, demand to all neighboring cells in $\mathscr{B}$ is scaled down with this factor Ψ_A. Now, the flow from A to B is set as the minimum of demand and supply. This flow is assumed to be constant between two consecutive time steps. The accumulation in any cell A towards destination s can now be updated based on the flows from B to A with destination s, indicated $q^A_{B,s}$ and the flow in the opposite direction, $q^B_{A,s}$:

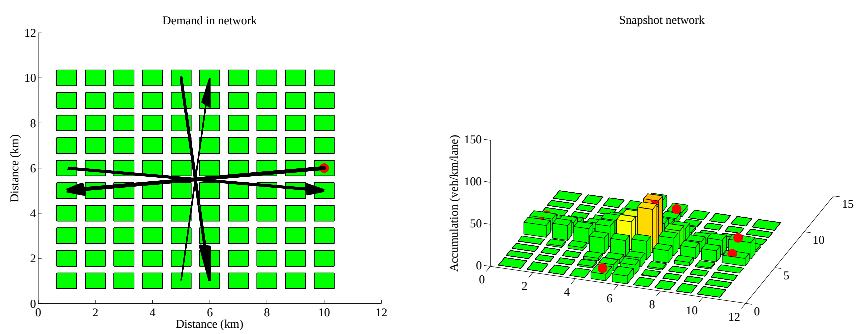
$$K^s_A(t+\tau) = K^s_A(t) + \left(\sum_{B \in \mathscr{B}} q^A_{B,s} - \sum_{B \in \mathscr{B}} q^B_{A,s} \right) \tau / w_A \tag{6}$$

From the flow, vehicles have to be translated into accumulation using the simulation time step τ and the road length w_A.

5 Application on a case study

For the case study we set up a network with 10x10 cells, each representing an area and all having the same characteristics. The cells have a size of 1x1 km and 10 kms of roadway length. The NFD of the cells is shown in figure 2(a). The capacity on the boundary between two cells is high enough that it does not restrict the flow. The time step used in the case study is 15 seconds.

A cross-network demand is loaded onto the network, shown graphically in figure 3(a). The arrow width indicates the size of the demand. The base demand for directions top-down and left-right is 625 veh/h, to left is 833 veh/h and the demand bottom-up is 312 veh/h. To this base demand, an extra demand is added, representing the loading onto the network, and then gradually reducing. After the demand has decreased to zero, the simulation continues to empty the network.



(a) Demands in the network (b) Case study network

Fig. 3 Case study

For the routing we use the Floyd Warshal algorithm [3]. Travel costs per cell are updated on-line, based on the time it costs to cross a cell. Routing variation is ensured by doing a probit assignment with 10% error in the perceived travel costs.

Figure 3(b) shows a snapshot of the model during the simulation. It shows that traffic is clustering around the middle cells, and due to the congestion there, traffic is taking alternative routes around the city center. This shows the model works in practice and shows plausible results.

6 Conclusions

This paper introduced the Network Transmission Model describing the traffic flow dynamics on an aggregate level. The network was splitted into cells and for the traffic flow dynamics a numerical approach based on the MFD was introduced. The model is face valid, but further studies should test the model and calibrate and validate it against real data or more often used traffic simulation programs. Once done, the model seems promising to test network traffic control using model predictive control.

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