Modelling route choice behaviour in a tolled road network with a time surplus maximisation bi-objective user equilibrium model

Judith Y. T. Wang[1], Matthias Ehrgott[2]

[1]School of Civil Engineering & Institute for Transport Studies, University of Leeds, U.K.
[2]Department of Management Science, Lancaster University, U.K.

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Overview

We are interested in modelling the route choice behaviour in a tolled road network . . .

There are essentially two conventional approaches in tolling analysis (Florian, 2006):

1. Models based on generalised cost path choice (UE)
2. Models based on explicit choice of tolled facilities (SUE)

(1) Models based on generalised cost path choice

Wardrop (1952) defined *user equilibrium* as:

"No user can improve his travel time by unilaterally changing routes"

Two key assumptions:

1. All users have the same objective, i.e. to minimise travel time or generalised cost
2. Users have perfect knowledge of the network, i.e. they know the travel times that would be encountered on all available routes between their origin and destination

Various ways have been applied to tackle these problems

Dial (1971) was the first to introduce a probabilistic assignment concept to address this problem:

1. The model gives all *efficient* paths between a given origin and destination a non-zero probability of use, while all inefficient paths have a probability of zero.
2. All efficient paths of equal length have an equal probability of use.
3. When there are two or more efficient paths of unequal length, the shorter has the higher probability of use.

(2) Models based on explicit choice of tolled facilities

Daganzo and Sheffi (1977) defined *stochastic user equilibrium* as:

"No user can improve his perceived travel time by unilaterally changing routes"

User's perceived travel time function on route k, $\tilde{T}_k$, has two components:

$$\tilde{T}_k = T_k + \epsilon_k$$

where
T_k is the systematic component
ϵ_k is an error term representing the random component

There are two classical SUE models

Depending on the assumption on the distribution of the error term, ϵ_k:

1. Gumbell distribution
   - Fisk (1980)'s logit-based model
2. Normal distribution
   - Sheffi and Powell (1982)'s probit model

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Why are we interested in modelling the route choice behaviour in a tolled road network?

1. Models based on generalised cost path choice (UE)
   - Users behave differently – they do not necessarily behave as if they have the same objective of minimising generalised cost
   - Multi-user class can only address this problem partially
   - Empirical evidence of non-linear effect of value of time (Hensher and Truong, 1985)
2. Models based on explicit choice of tolled facilities (SUE)
   - Logit model has the independence of irrelevant alternatives (IIA) property
   - Probit model requires intensive computational effort for Monte Carlo simulation

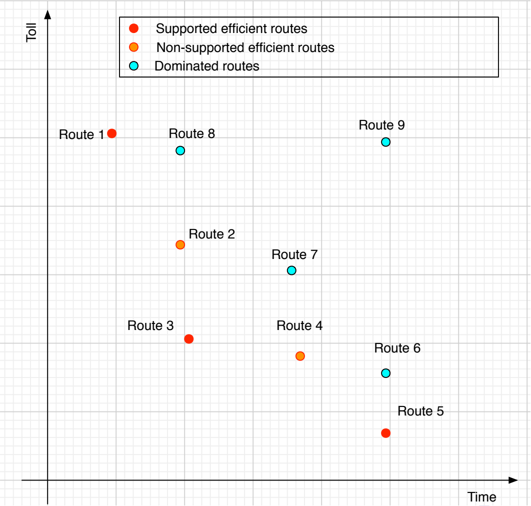
What is bi-objective traffic assignment?

- Dial (1979) is the first to introduce bi-objective in traffic assignment
- All users have two objectives:
  1. minimise toll cost
  2. minimise travel time
- However, Dial (1979) combined the two objectives into a single objective:

$$C_k(\mathbf{F}) = M_k(\mathbf{F}) + \alpha T_k(\mathbf{F})$$

  where $M_k(\mathbf{F})$ is the monetary cost associated with path k; α is a value of time that follows a probability distribution, that converts the travel time $T_k(\mathbf{F})$ into a monetary value

What we might have missed with single objective?



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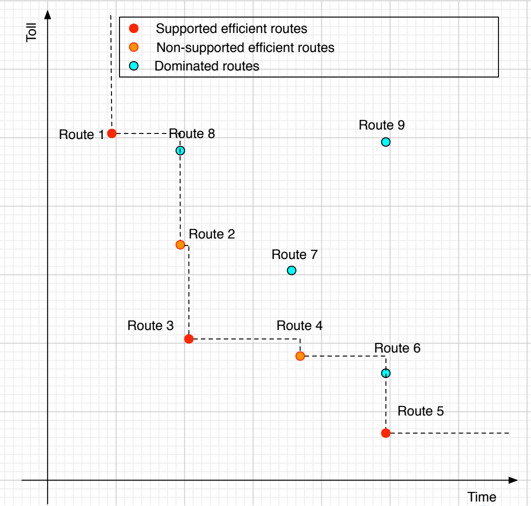
Selecting efficient routes



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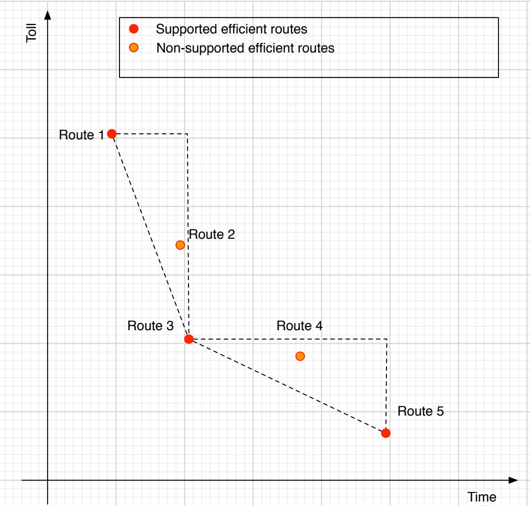
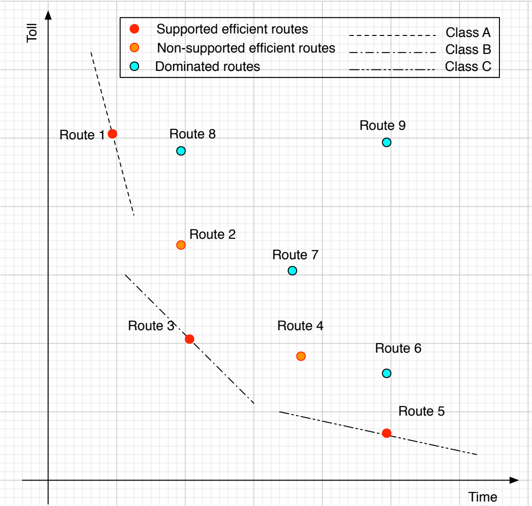
Supported vs non-supported efficient routes



What might be missed in Dial (1979)'s model?



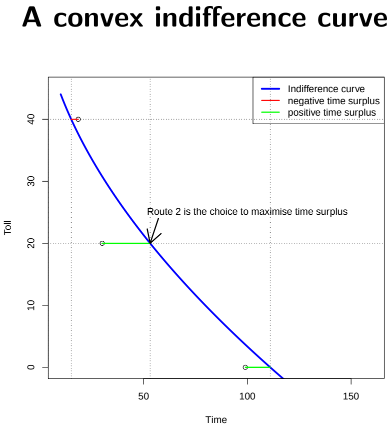
Overview

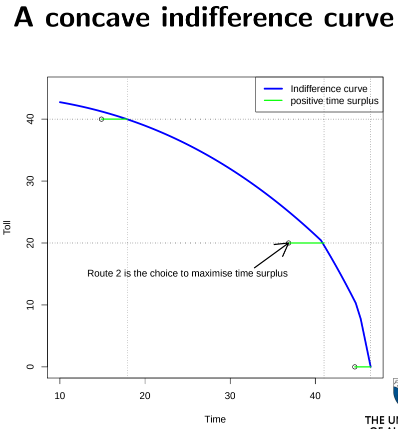
Bi-objective Traffic Assignment (BUE)

- All users have two objectives:
  1. minimise travel time
  2. minimise toll cost
- Wang *et al.* (2010) define *bi-objective user equilibrium* (BUE) as follows:

"Under *bi-objective user equilibrium* conditions traffic arranges itself in such a way that no individual trip maker can improve either his/her toll or travel time or both without worsening the other objective by unilaterally switching routes"

The Time Surplus Maximisation Concept





The Time Surplus Maximisation Concept

- Given the indifference curves T_p^{max} for all $p \in Z$, we define time surplus for path $k \in K_p$ as

$$TS_k(\mathbf{F}) := T_p^{max}(\tau_k) - T_k(\mathbf{f})$$

- All users have the same objective: to maximise time surplus

$$k^* = \mathrm{argmax}\{TS_k(\mathbf{F}) : k \in K_p\}$$

- Each user might have his/her own indifference curve

The Time Surplus Maximisation BUE Condition (TSmaxBUE)

"Under the *Time Surplus Maximisation condition* traffic arranges itself in such a way that no individual trip maker can improve his/her time surplus by unilaterally switching routes"

or alternatively

"Under the *Time Surplus Maximisation condition* all individuals are travelling on the path with the highest time surplus value among all the efficient paths between each O-D pair"

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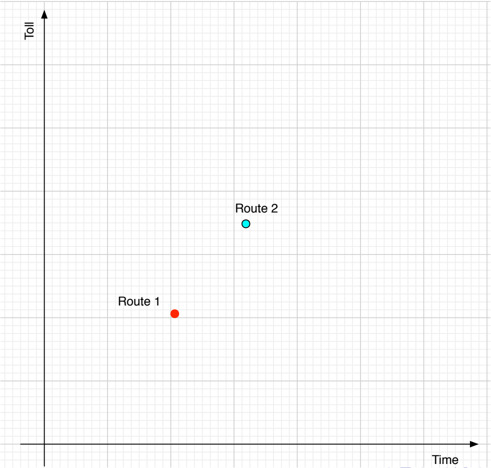
Overview

How do TSmaxBUE and BUE relate to each other? TSmaxBUE $\Longrightarrow$ BUE

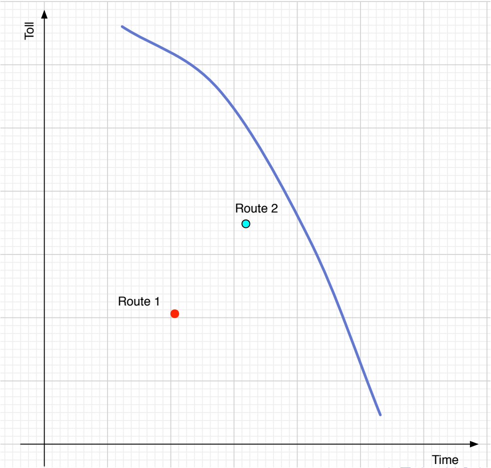
Theorem 2

Let $G = (N, A)$ be a network, $Z \subset N \times N$ be a set of O-D pairs with demand $D_p > 0$ for all $p \in Z$. Let τ_a denote the toll of link a and $t_a(f_a)$ be the travel time function of link a. Assume that $\mathbf{F}^*$ is a TSmaxBUE flow. Then $\mathbf{F}^*$ is also a bi-objective equilibrium flow with respect to the objectives $C^{(1)}(\mathbf{F}) = T_k(\mathbf{f})$ and $C^{(2)}(\mathbf{F}) = \tau_k$.

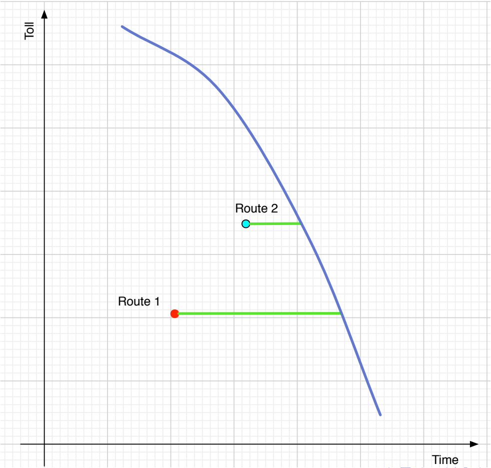
Let's say the TSmaxBUE solution is not a BUE solution, both Routes 1 and 2 have positive flow



For any strictly decreasing indifference curve, for example this one ...



That is not possible! Route 2 should not have positive flow! $\therefore$ TSmaxBUE $\Longrightarrow$ BUE



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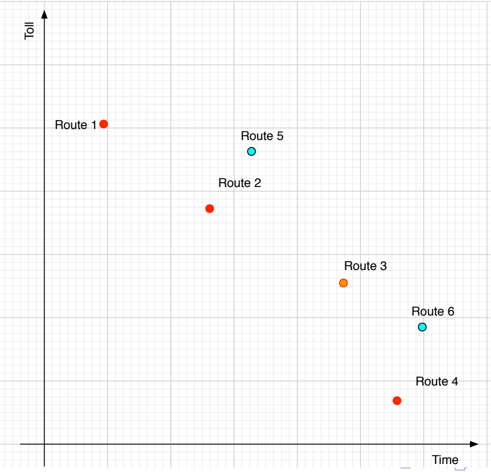
How do TSmaxBUE and BUE relate to each other? TSmaxBUE $\Longleftarrow$ BUE

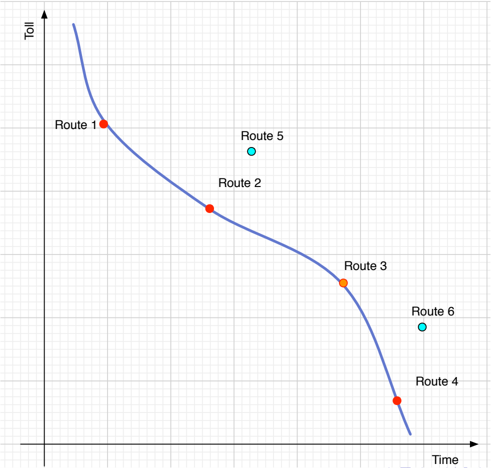
Theorem 3

Let $G = (N, A)$ be a network, $Z \subset N \times N$ be a set of O-D pairs with demand $D_p > 0$ for all $p \in Z$. Let τ_a denote the toll of link a and $t_a(f_a)$ be the travel time function of link a. Assume that $\mathbf{F}^*$ is a bi-objective equilibrium flow, with respect to objectives $C^{(1)}(\mathbf{F})$ and $C^{(2)}(\mathbf{F})$ as in Theorem 2. Then there exists an indifference function T^{max} such that $\mathbf{F}^*$ is also a TSmaxBUE flow.

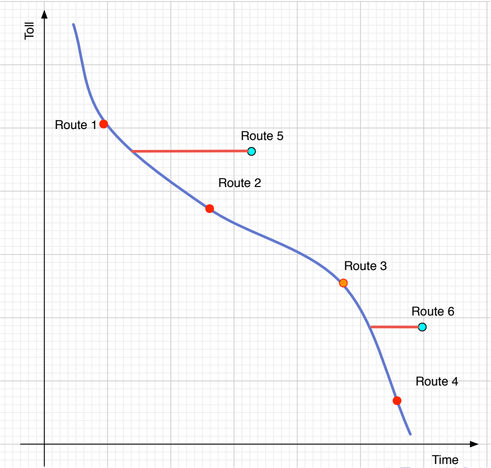
Let's say we have a BUE solution, efficient Routes 1 to 4 have positive flow, all dominated routes have zero flow



To show that this is a TSmaxBUE, we construct an indifference curve



Every efficient route has zero time surplus while every dominated route has negative time surplus!

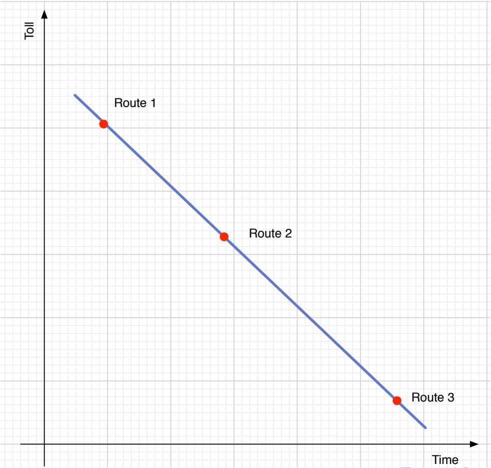


How do UE and TSmaxBUE relate to each other? Generalised Cost UE $\Longrightarrow$ TSmaxBUE

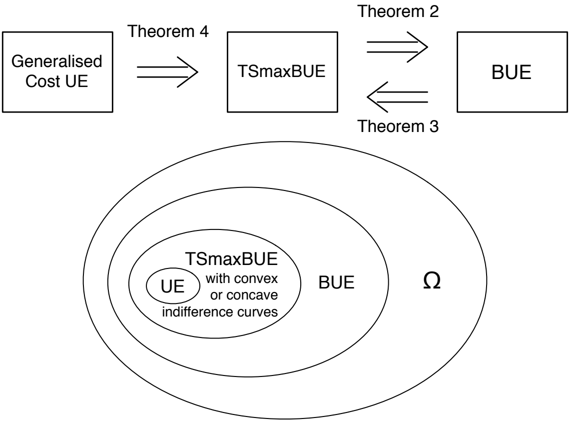
Theorem 4

Let $G = (N, A)$ be a network, $Z \subset N \times N$ be a set of O-D pairs with demand $D_p > 0$ for all $p \in Z$. Let τ_a denote the toll of link a and $t_a(f_a)$ be the travel time function of link a. Assume that $\mathbf{F}^*$ is an equilibrium flow with respect to the generalised cost objective $C(\mathbf{F}) = \tau_k + \alpha T_k(\mathbf{f})$. Then there exists an indifference curve T^{max} such that $\mathbf{F}^*$ is also a TSmaxBUE flow.

A UE solution with value of time α is TSmaxBUE solution with a straight-line indifference curve with slope $-1/\alpha$



The relationship between equilibrium concepts



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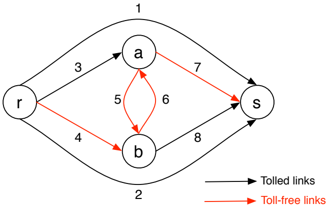
Overview

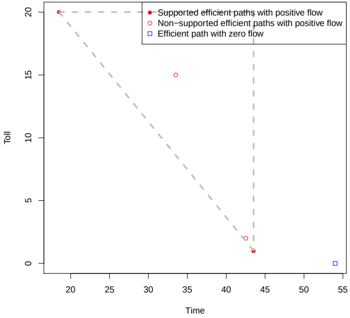
Example 1 – A Four Node Network

Route	Path	Length	Free-flow Travel Time	Toll	Max Time
1	1	30	18.0	20	25
2	2	30	22.5	15	40
3	3 – 7	30	36.0	1	50
4	4 – 8	30	36.0	1	50
5	3 – 5 – 8	22	26.4	2	49
6	4 – 6 – 7	45	54.0	0	51

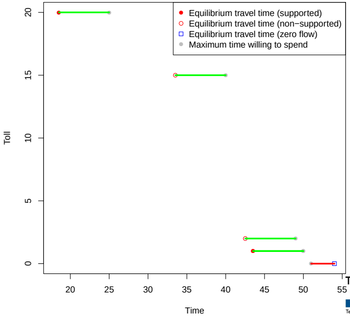


TSmaxBUE is indeed more general than generalised cost user equilibrium

Efficient paths do not all optimise generalised cost



Time surplus on used paths are equal and maximal



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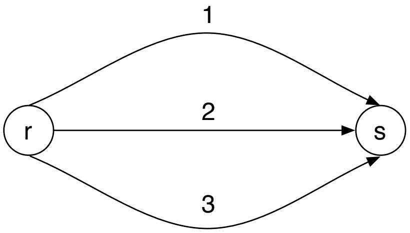
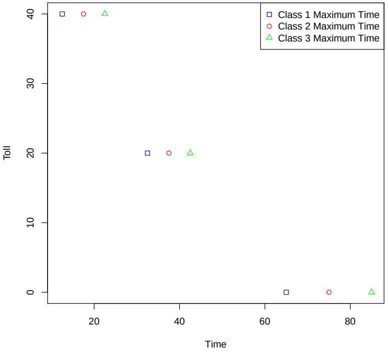
Example 2 – A Three Link Network

Route	Type	Distance (km)	v_0 (km/hr)	t_0 (mins)	Capacity (veh/hr)
1	Expressway	20	100	12	4000
2	Highway	50	100	30	5400
3	Arterial	40	60	40	4800



Maximum Time Willing to Spend by User Class



Route k	Class 1 t_1^{max}	Class 2 t_2^{max}	Class 3 t_3^{max}
1	12.5	17.5	22.5
2	32.5	37.5	42.5
3	65.0	75.0	85.0

Multiple user class test parameters for UE & SUE

Parameter	Class 1	Class 2	Class 3
VOT in UE & SUE	\$3	\$2	\$1
θ in SUE	0.1	0.1	0.1
Demand	5000 veh/h	5000 veh/h	5000 veh/h

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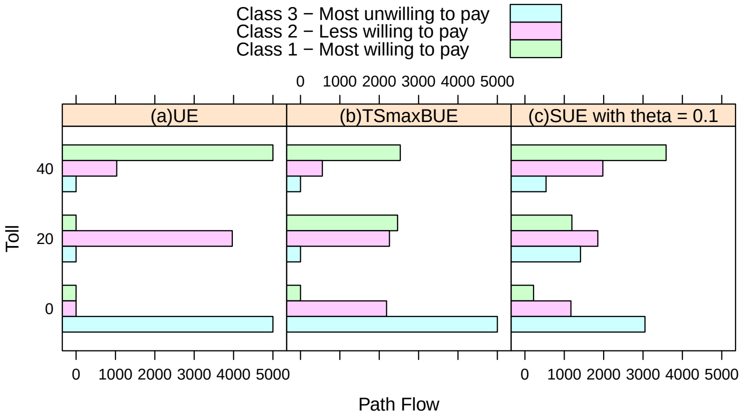
Path Flow by User Class



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Conclusions

- We propose a BUE model *without combining the two objectives* of minimising toll and travel time
- It is *important not to assume toll and travel time are additive*, because some rational route choices must have zero flows
- We introduce *indifference curves* to model user preferences
- With indifference curves, we can model the *trade off between toll and time* in a two-dimensional space with no restrictions
- *Non-linear effect* of value of time can be modelled

Conclusions

- We introduce the concept of *time surplus*, defined as the maximum time a user willing to spend minus the actual time
- Under the TSmaxBUE condition, all individuals are travelling on *an efficient path with maximal time surplus*
- We show that TSmaxBUE is a *proper generalisation* of generalised cost user equilibrium
- We prove that the TSmaxBUE condition is *equivalent* to bi-objective user equilibrium

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Some Further Research Topics

- Consideration of different combinations of the three most important factors affecting route choice behaviour:
  1. travel time
  2. travel time reliability
  3. monetary cost (toll)
- Efficient algorithms to solve the TSmaxBUE model for realistic networks
- Consideration of elastic demand
- Policy analysis with a bilevel multi-objective approach

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