A hybrid implementation mechanism of tradable network permits system:

An auction mechanism with day-to-day capacity control

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Introduction

- Congestion pricing (price-based regulation)
  - working effectively when a road manager can calculate an optimal toll level
  - needing **accurate** and **detailed demand information**
    - e.g., OD demands, VOT, desired arrival/departure time

- **Asymmetric information**
  - between the manager and road users
  - very difficult to obtain such private information
    - distorting toll levels
    - resulting in economic losses
What is tradable network permits scheme?

- ** Tradable Network Permits (TNP)** [Akamatsu et al. (06, 07)]
  - a right that allows a permit holder to pass through specific bottleneck during pre-specified time period
  - quantity-based regulation
    e.g., highway booking

- **Trading market** for network permits
  - time-dependent
  - freedom of permit choice
  - Allocation and prices are determined through the markets.
**Why is the TNP scheme needed?**

- **No queuing congestion**
  - # of permits of each link ≤ bottleneck capacity

- **No detailed information on user demand**
  - manager only needs to know bottleneck capacity.
    - cf. congestion pricing (or price-based regulation)

- **Efficiency in general networks** [Akamatsu, (07)]
  - equilibrium resource allocation pattern
    - = Dynamic System Optimal (DSO) assignment
      - assumption: trading markets are perfectly competitive,
        i.e., trading processes were treated as a black box.
How to implement the TNP scheme?

- Markets do what they are supposed to do, however, **only if they are well structured.** [McMillan (02)]
  - preventing **manipulating** prices
  - achieving an efficient allocation

- Auction mechanism for a bottleneck [Wada & Akamatsu (10)]
  - Vickrey-Clarke-Groves (VCG) mechanism
    - **strategy-proof** (i.e., truthful bidding is a dominant strategy)
    - **Permits allocation pattern is efficient.**

- General networks
  - A naïve formulation of the problem leads to **NP-hardness** owing to the complex relationship between link and path.
Purpose of the study

- A novel auction mechanism to implement TNP scheme for general networks
  - enabling each user to purchase the preferred bundle of permits (path)
  - achieving a DSO allocation of network permits in a computationally efficient manner

- Approach
  - evolutionary (day-to-day) approach
    - a path-based auction with day-to-day capacity control
  - obviating path enumeration by introducing a column generation procedure
Tradable permits for managing traffic congestion

- Quantity-based regulation + market institution
  - possibilities of using tradable permits for managing congestion [Verhoef et al., (97)]
  - none describes **time-dependent tradable permits** for eliminating **bottleneck congestion**.

- Tradable travel credit scheme [Yang and Wang (11)]
  - is superficially similar to but **fundamentally different** from tradable network permits scheme
  - is not a quantity-based regulation for **directly** reducing congestion but rather a **redistribution scheme of income**
Setting

- A discrete time DTA on general networks with multiple Origin-Destination (OD) pairs
  - point (or physical) queue model
  - \( \mu_a \) - bottleneck capacity = # of permits of link \( a \)
    - no queuing congestion

- Behavior assumption for atomic road user
  - at most single trip per day between an OD
  - utility max.: choice rule of path and destination arrival time
  - must purchase a bundle of permits corresponding to a path
User valuation and utility

- User i’s valuation for path r and destination arrival time t
  - private information
  - including constant travel time
    \[ v_{i,r}(t) = [v_i]_{r,t} \geq 0 \]

- Quasi-linear utility = valuation − permit purchase cost
  \[ u_{i,r}(t) = v_{i,r}(t) - p_r(t) \]

- Allocation of bundles of permits (paths)
  \[ f_{i,r}(t) = [f_i]_{r,t} \in \{0, 1\} \]
  \[ \sum_{r,t} f_{i,r} \leq 1 \]
**DSO allocation of network permits**

- **Dynamic system optimal problem [DSO] (atomic user)**

\[
\max \{f_i\} \sum_i v_i \cdot f_i \quad \text{... social surplus}
\]

- subject to

\[
1 \cdot f_i \leq 1 \quad \forall i \quad \text{... at most single trip per day}
\]

\[
\sum_i x_i \leq \mu \quad \text{... link capacity constraint}
\]

\[
x_i = \Delta_i f_i \quad \forall i \quad \text{... relationship path and link allocation}
\]

- allocation of permits (Links): \( x_{i,a}(t) = [x_i]_{a,t} \in \{0, 1\} \)
- path-link incidence matrix: \( \Delta_i \)
Difficulties of solving the problem [DSO]

- **Incomplete information** on the objective function
  - **Users’ private valuations** are unknown

  ✓ Vickrey-Clarke-Groves (VCG) combinatorial auction
    - gives users an incentive to report their valuations truthfully
    - must solve [DSO] **exactly** in many times to calculate allocation and **individual payment** (i.e., marginal cost)

- [DSO] is integer multi-commodity flow problem
  - complex relationship between **link and path variables** on **individual allocations**
  - **NP-hard**: no polynomial time algorithm exists
Basic ideas of day-to-day auction mechanism

- Decomposition of [DSO] based on Benders’ method
  - master-P – adjusting # of bundles of permits (path capacity) on day-to-day basis
  - sub-P – assigning bundles (path capacity) to users

- Solve the sub-P by an auction mechanism
  - gives users an incentive to report their valuation truthfully
  - can obtain the efficient permit allocation with incomplete information

- Assumption: users try to maximize their current utilities, i.e., (myopic) best response dynamics
Reformulation of the problem [DSO]

- Introducing **non-individual** integer variables
  - $F$ - aggregated path variables (path-capacities)
  - $X$ - aggregated link variables
- Equivalent optimization problem to the problem [DSO]

\[
\max_{\{F_w\},\{f_i\}} \sum_i v_i \cdot f_i \quad \text{... social surplus}
\]

- subject to
  - \[1 \cdot f_i \leq 1 \quad \forall i \quad \text{... at most single trip per day}\]
  - \[\sum_i f_i \leq F_w \quad \forall w \quad \text{... Path capacity constraint for OD pair w}\]
  - $X \leq \mu \quad \text{... link capacity constraint}$
  - \[X = \sum_w \Delta_w F_w \quad \text{... relationship path and link allocation}\]
Reformulation of the problem [DSO]

- Introduce *non-individual* integer variables
  - $\mathbf{F}$ - non-individual path variables (path-capacities)
  - $\mathbf{X}$ - non-individual link variables

- Equivalent optimization problem to the problem [DSO]

$$\max_{\{F_w\},\{f_i\}} \sum_i \nu_i \cdot f_i$$

- subject to

  $1 \cdot f_i \leq 1 \quad \forall i$

  $\sum_i f_i \leq F_w \quad \forall w$

  $\mathbf{X} \leq \mu$

  $\mathbf{X} = \sum_w \Delta_w F_w$

*all constrains for individual variables are represented by path variables.*
Framework of the day-to-day auction mechanism

master-P: $F$
path capacity adjustment phase

sub-P: $f_i$
auction phase

sub-P: $f_i$
auction phase

sub-P: $f_i$
auction phase
Decomposition of the problem [DSO]

- **Sub-problem (assignment problem): auction phase**
  - assigning bundles to users for **fixed path capacities**
  - satisfying **totally unimodularity**, i.e., \( \text{LP} = \text{IP} \)

\[
\max_{\{F_w\}} \sum_w S_w(F_w) = \max_{\{f_i\}} \sum_i v_i \cdot f_i
\]

subject to

\[
X \leq \mu
\]

\[
X = \sum_w \Delta_w F_w
\]

subject to

\[
1 \cdot f_i \leq 1 \quad \forall i
\]

\[
\sum_i f_i \leq F_w
\]

unknown variables: individual path variables
Decomposition of the problem [DSO]

- Dual sub-problem: auction phase
  - providing information on prices/payoffs
  - $\pi_i$: user $i$’s payoff, $p$: bundles prices

\[
\max_{\{F_w\}} \sum_w S_w(F_w) = \min_{\{p_w\},\{\pi_i\}} \sum_i \pi_i + p_w \cdot F_w
\]
subject to
\[
\pi_i 1 \geq v_i - p_w \quad \forall i
\]

unknown variables: payoff / price variables

\[
X \leq \mu
\]

\[
X = \sum_w \Delta_w F_w
\]
Decomposition of the problem [DSO]

- **Master problem: path capacity adjusting phase**
  - adjusting path capacities based on **demand information**
    (payoffs and prices) of each day auction phase

\[
\max_{\{F_w\}} \sum_w S_w(F_w) = \min_\sigma \left[ \sum_i \pi_i^0 + p_w^0 \cdot F_w \right]
\]

- subject to

\[
X \leq \mu \\
X = \sum_w \Delta_w F_w
\]

\(\sigma\): set of all extreme points of dual constraints

*unknown variables:* non-individual variables

- using relaxation: generating extreme points iteratively
- using heuristics: linear relaxation, box-step constraints
Procedure of day-to-day auction mechanism

**Day 1**

Master problem: Path capacity adjusting

Initial path capacity $F^1$

Subproblem: Auction

Dual: Price $p^1$, Payoff $\pi^1$

Primal: Allocation $f^1$

**Day 2**

Master problem: Path capacity adjusting

Path capacity $F^2$

Subproblem: Auction

Dual: Price $p^2$, Payoff $\pi^2$

Primal: Allocation $f^2$
Obtaining truthful users’ valuation

- **Ascending proxy auction** [Demange et al. (86), Parkes & Ungar (02)]
  - users report their valuations to proxy agents
  - agents bid most preferred items under the current prices
  - choosing the overdemanded set & raising prices in the set
    - **Individual rationality**: winners’ payoffs are non-negative
    - **strategy-proof**: honesty is a best strategy for users
    - **efficiency**: Pareto efficient allocation can be achieved

- Corresponding to the mathematical programming
  - the process of the auction is corresponding to the *primal-dual (Hungarian) algorithm* for solving the sub-problem
Properties of the day-to-day auction mechanism

- **Proposition 1 (Convergence property)**
  The day-to-day auction mechanism converges in a finite number of iterations.

- **Proposition 2 (Efficiency property)**
  The permits allocation pattern by the mechanism converges to the most efficient allocation when the number of users is large.

proof:
\[
\lim_{N \to \infty} \frac{S_L - S_I}{S_L} = 0
\]

- $S_L$ : obj. fun. of linear relaxation of MP
- $S_I$ : obj. fun. of integer MP
Introducing **column generation** procedure

1. Considering a **subset of paths** of [DSO]. For the fixed path set, executing the day-to-day auction mechanism.

2. After convergence, **each user generates** preferred path based on the previous day-to-day auction. Go to Step 1.

- Path generation is efficient because **the numerous # of users generate paths simultaneously**
Numerical example: convergence process

- Sioux Falls network with 528 OD pairs
- the total number of users is 90150

![Graph showing ratio of realized value of SS to the maximum value over time. The graph illustrates two phases: The path generation phase and The day-to-day auction phase. Additionally, there are annotations pointing out MP’s objective function value (upper bound) and Realized value of social surplus (sub-problem).]
Conclusion

- We proposed the day-to-day auction mechanism for implementing tradable network permits scheme for general networks.

- We showed that the mechanism has desirable properties: strategy-proof; finite convergence; Pareto efficiency (DSO).

- We extended the mechanism to obviate path enumeration by introducing a column generation procedure.
Thank you for your attention

My first baby (son) was born yesterday!!

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References


**References**

Future works

- Applying the proposed mechanism to the management of other transportation networks
  - railway networks, freight networks, logistics networks etc.
  - Problem: a network manager aims to maximizes not the social surplus but his/her profit

- Tradable network permits under the second-best situation
  - Queuing congestion occurs at a link that is not controlled.
  - needing connect the tradable network permits scheme to a DTA problem.
imagine that ...

Each user’s car has “agent software”

Agent software

Agent

Bottleneck

General networks
Multi-agent system

Each agent chooses a path and arrival time using **local information only**

**Agent software**

**Agent**

**General networks**

**Bottleneck**
Each agent deals with the cumbersome procedure of trading the bottleneck permits.
Feasibilities for implementation

- Technical point of views
  - Procedures for network permits
    » Dedicated short range communication (DSRC) system (e.g., Electric toll collection (ETC))
  - Trading markets
    » Internet auction markets (inexpensive!!)

- Institutional point of views
  - Minimal legal restrictions is needed
Assigning schemes of network permits

- Market selling scheme (In this study)
  - The road manager **sells** all the bottleneck permits to users in the trading market

- Free distribution scheme
  - The road manager distributes all the permits to users **for free** according to methods that consider the **equity among users**, e.g., rotation system of license plate numbers

- Remark
  - In terms of the efficiency of resource allocation, the two schemes are essentially identical.
Tradable network permits with stochastic arrivals

- Some users arrive at a bottleneck late (or fast)

- Stochastic queuing congestion can be decreased when a number of permits for the link is less than the capacity
  - about 80% [Kasahara & Akamatsu (06)]