Modelling Supported Driving as an Optimal Control Cycle: Framework and Model Characteristics

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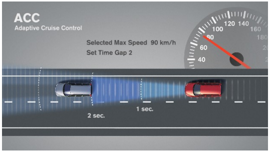
Outline

Background

- Global research interests in Advanced Driver Assistance Systems and Cooperative Systems
- Increase of Adaptive Cruise Control (ACC) systems equipped vehicles on road
- ACC system automates the longitudinal driving tasks:
  - Cruising mode: maintain free speed
  - Following mode: maintain desired gap



- Induced changes in individual vehicular behaviour and collective traffic flow operations

Existing ACC contollers (or models)

- Widely-used Constant Time Gap policy, i.e. Helly car-following model without time delay
- Overruled by drivers at highly non-stationary conditions and congestions
- Difficult to incorporate cooperative driving concept with Vehicle-Vehicle (V2V) communications
- Unable to fulfil multiple objectives, e.g. maximising safety, efficiency and sustainability

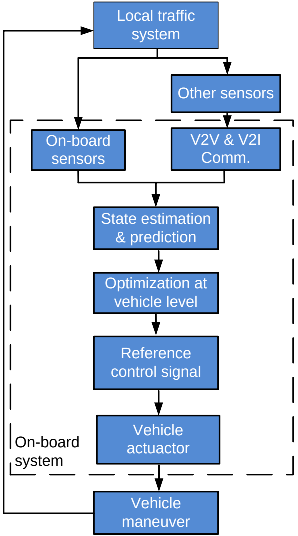
This contribution

- An optimal control framework for driver assistance and cooperative systems
- An ACC controller with human-like behaviour
- A Cooperative ACC (C-ACC) controller that captures vehicle-vehicle collaboration
- New insights into stability characteristics of the controllers

Control assumptions

- Controlled acceleration, i.e. automatic control of throttle and brake pedal
- Information of other vehicles influencing control decisions available
- No delay in the control loop
- Deterministic case

Rolling horizon approach



Cost minimisation in one control cycle

$$\mathbf{u}^* = \arg\min J(\mathbf{x}, \mathbf{u}|\mathbf{x}_0)$$

$$J(\mathbf{x}, \mathbf{u}|\mathbf{x}_0) = \int_{t_0}^{\infty} e^{-\eta\tau} \mathcal{L}(\mathbf{x}, \mathbf{u}) d\tau$$

s.t.

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \mathbf{x}_0 = \mathbf{x}(t_0)$$

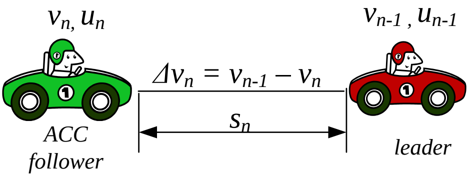
- $\mathbf{x}$: system state; $\mathcal{L}$: running cost
- $\eta > 0$: discount factor, cost discounted in the (uncertain) future and decreases *exponentially* after a horizon $1/\eta$
- $\mathbf{u}^*$: optimal acceleration, can be found by *Dynamic Programming* approach

Controller design procedure under the framework

1. Define system state and determine state prediction model (i.e. system dynamics equation)
2. Specify cost function under control objectives
3. Find optimal acceleration
4. Verify the controller performance

Example 1: ACC controller

System state: $\mathbf{x} = (s_n, \Delta v_n, v_n)'$



State prediction model: system dynamics equation

$$\dot{\mathbf{x}} = \left(\Delta v_n, u_{n-1} - u_n, u_n \right)'$$

ACC control objectives

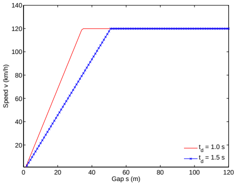
- Maximise safety in following mode, by penalising approaching leader
- Maximise travel efficiency, by penalising deviation from desired speed or free speed
- Maximise driving comfort, by penalising large acceleration/deceleration

$$\mathcal{L} = \begin{cases} \underbrace{c_1 e^{\frac{s_0}{s}} \Delta v^2 \cdot \Theta(\Delta v)}_{\textit{safety}} + \underbrace{c_2 (v_d(s) - v)^2}_{\textit{efficiency}} + \underbrace{\frac{1}{2} u^2}_{\textit{comfort}} & \text{following} \\ \underbrace{c_3 (v_0 - v)^2}_{\textit{efficiency}} + \underbrace{\frac{1}{2} u^2}_{\textit{comfort}} & \text{cruising} \end{cases}$$

Optimal acceleration of ACC vehicle

$$u^*_{\text{ACC}} = \begin{cases} \underbrace{\frac{2c_1 e^{\frac{s_0}{s}}}{\eta}\left(\Delta v - \frac{s_0 \Delta v^2}{\eta s^2}\right) \cdot \Theta(\Delta v)}_{\textit{decelerate when approaching}} + \underbrace{\frac{2c_2}{\eta}\left(1 + \frac{2}{\eta t_d}\right)(v_d(s) - v)}_{\textit{match desired speed}} & \text{following} \\ \underbrace{\frac{2c_3}{\eta}(v_0 - v)}_{\textit{match free speed}} & \text{cruising} \end{cases}$$

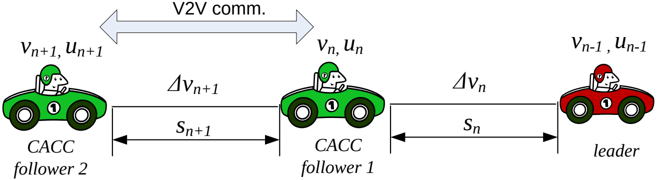
Desired speed v_d as a function of gap:



Satisfies necessary conditions for plausible car-following models!

Example 2: Cooperative ACC (C-ACC)

- Two CACC follower exchange information (gap and relative speed) via V2V communication
- Negotiate and coordinate their (car-following) behaviour under a common objective
- System state $\mathbf{x} = (s_n, \Delta v_n, v_n, s_{n+1}, \Delta v_{n+1}, v_{n+1})'$
- State prediction model:
  $\dot{\mathbf{x}} = (\Delta v_n, u_{n-1} - u_n, u_n, \Delta v_{n+1}, u_n - u_{n+1}, u_{n+1})'$



C-ACC cost and acceleration

- Control objective: maximising safety, efficiency and driving comfort for both followers
- Joint cost function: sum of costs for two followers

$$u^*_{\text{C-ACC}} = \underbrace{u^*_{\text{ACC}}}_{\text{ACC acceleration}} \underbrace{- \frac{2c_1 e^{\frac{s_0}{s_{n+1}}}}{\eta} \left(\Delta v_{n+1} - \frac{s_0 \Delta v^2_{n+1}}{2\eta s^2_{n+1}} \right) \cdot \Theta(\Delta v_{n+1})}_{\text{accelerate when } \Delta v_{n+1} < 0} \underbrace{- \frac{2c_2}{\eta^2 t_d} (v_d(s_{n+1}) - v_{n+1}}_{\text{decelerate when } v_{n+1} < v_d(s_{n+1})})$$

Anlytical framework for stability analysis

- Consider a generalised acceleration function $u(s, \Delta v, v, s_b, \Delta v_b, v_b)$, with $s_b, \Delta v_b, v_b$ denoting the situation behind
- Find equilibrium gap-speed relation $v_e(s_e)$ by setting $u = 0$ and $\Delta v = 0$
- Insert small disturbances of gap and speed to a vehicle at equilibrium
- Take derivatives of disturbance and get disturbance dynamical equation (DDE)
- Solve the DDE and find the signs of the roots using Fourier analysis

String stability criteria

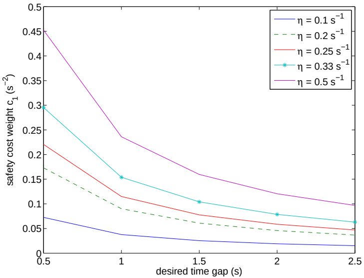
ACC controller:

$$v_e'(s_e)^2 \leq v_e'(s_e) u_{\Delta v} + \frac{u_s}{2}$$

C-ACC controller:

$$v_e'(s_e)^2 \leq v_e'(s_e) u_{\Delta v} + \frac{u_s}{2} + \underbrace{v_e'(s_e)\left(u_{\Delta v_b} - u_{v_b}\right) - \frac{u_{s_b}}{2}}_{\text{stabilisation effects of C-ACC}}$$

Neutral string staility line



String stability of ACC platoon is enhanced with:

- larger safety cost weight, more anticipative driving style
- larger time gap, larger following distance
- smaller discount factor, longer prediction horizon

Convective and absolute instability

If string instability prevails, in which direction the disturbances propagate in the x-t plane: upstream, downstream or both?

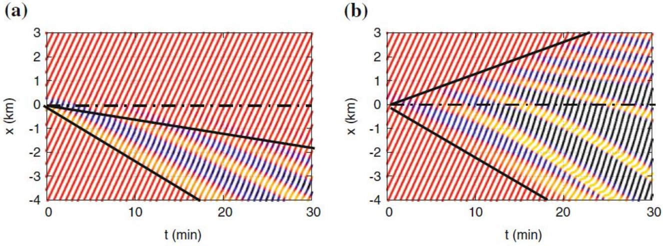


Figure: (a) Convective upstream instability; (b) Absolute instability (Treiber and Kesting, 2013).

Stability diagram using Fourier transform

S: Stability; U: convective Upstream instability;
A: Absolute instability; D: convective Downstream instability

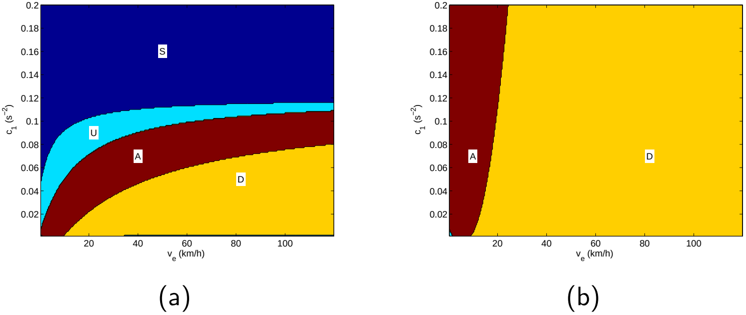


Figure: (a) ACC controller; (b) C-ACC controller.

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Summary

- An optimal control framework for driver assistance and cooperative systems
- An ACC controller with plausible car-following behaviour
- A C-ACC controller under collaborative driving concept
- Rigorous stability criteria for ACC and C-ACC can be used as guidance for controller design and tuning
- The C-ACC controller produces significantly different string instability property compared to the ACC controller

Future research

- Including delay and inaccuracy in the framework
- Design cooperative vehicle controller to improve stability
- Challenge to model-based traffic state estimation, prediction and control methods in Cooperative Systems

Further reading

- M. Treiber and A. Kesting. Traffic Flow Dynamics. Springer, 2013.
- M. Treiber and A. Kesting. Evidence of Convective Instability in Congested Traffic Flow: A Systematic Empirical and Theoretical Investigation. Transportation Research Part B: Methodological, 2011, 45, 1362-1377
- J. A. Ward and R. E. Wilson. Criteria for convective versus absolute string instability in car-following models. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science, 2011.
- M. Wang, W. Daamen, S.P. Hoogendoorn, and B. van Arem. Rolling horizon control of intelligent vehicles with driver assistance and cooperative systems. Part I: Framework, numerical solution and non-cooperative system. Transportation Research Part C: Emerging Technologies, 2013 (Under review).
- M. Wang, W. Daamen, S.P. Hoogendoorn, and B. van Arem. Rolling horizon control of intelligent vehicles with driver assistance and cooperative systems. Part II: Applications on cooperative sensing and cooperative control. Transportation Research Part C: Emerging Technologies, 2013 (Under review).

Optimal control by cost minimisation

$$\mathbf{u}^* = \arg\min J(\mathbf{x}, \mathbf{u}|\mathbf{x}_0)$$

$$J(\mathbf{x}, \mathbf{u}|\mathbf{x}_0) = \int_{t_0}^{t_0+T} e^{-\eta\tau}\mathcal{L}(\mathbf{x}, \mathbf{u})d\tau + e^{-\eta(t_0+T)}\phi(\mathbf{x}(t_0+T))$$

s.t.

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \mathbf{x}_0 = \mathbf{x}(t_0)$$

- $\mathbf{u}^*$: optimal controlled acceleration in $[t_0, t_0 + T)$
- $\mathbf{x}$: local traffic system state
- $\mathcal{L}$: running cost; ϕ: terminal cost
- $\eta \geq 0$: discount factor, cost discounted in the (uncertain) future and decreases *exponentially* after a horizon $1/\eta$

Alternative solution approach for more general cost functions:

Characteristic velocity of waves

- Group velocity (v_g): signal waves propagates v_g
- Phase velocity (v_ϕ): center of the perturbation waves propagate with v_ϕ
- Signal velocity (c_s+ and c_s-): boundaries of the instability region

Calculation characteristic velocity

1. Partition initial perturbation $U(x,0)$ into linear waves with Dirac delta function.
2. Perform Fourier transform from physical space $U(x,0)$ to Fourier space $U(k,0)$. In Fourier space, all relevant perturbation modes are the same and equals unity.
3. Perform an inverse Fourier transform from Fourier space to physical space to get the complex perturbation amplitude $\tilde{U}(x,t)$.
4. Expand root λ around the wave number k_0 with the maximum growth rate with Taylor series.
5. Solve a well-defined Gaussian integral and take the real part to get spatio-temporal evolution of initial perturbation $U(x,t)$.
6. Find v_g, v_ϕ, c_{s+} and c_{s-}.

An alternative method with Laplace transform in Ward and Wilson (2011).