A CONTINUUM MODELING APPROACH FOR NETWORK VULNERABILITY ANALYSIS AT REGIONAL SCALE

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ABSTRACT

This paper presents an application of the continuum traffic equilibrium model for network vulnerability analysis that aims to resolve the critical issues faced by the network-modeling framework. In this study, a bi-level model is set up for finding the most vulnerable location(s) in the study region. At the lower-level model, a set of differential equations is constructed to describe the traffic equilibrium problem under capacity degradation. In the upper-level model, a constrained minimization problem is set up to find the most vulnerable location(s) that minimizes the accessibility index of the study region. A sensitivity-based solution algorithm that adopts the finite element method (FEM) is proposed to solve the bi-level model.

BACKGROUND

- **Vulnerability analysis**, which aims at identifying the weak spots in the transport network and the corresponding impacts upon failure, is vital in strategic planning to **identify the critical areas/roads for network improvements**.
- The traditional vulnerability analysis in **network-modeling based framework** faced the following **three critical issues**:

(i) Minor roads may not be included in the network coding. The omission of minor roads may lead to an overestimation of the importance of some major links due to the lack of alternative routes.

(ii) Demand locations are arbitrarily defined by zone centroids and centroid connectors. Due to the discrete representation of demand distribution, a certain link in the network may be attached to a particular demand location/group, and, hence, the failure of this link will cause a major impact to these travelers.

(iii) Link-wise failure in the current network-modeling based framework. The catastrophic disruptions in transport networks usually involved different types of wide-area natural disasters (e.g. flooding, earthquake) that could not be precisely represented by the network modeling framework.

- In this paper, a **continuum traffic equilibrium model** is proposed for vulnerability analysis. A **bi-level model**, which is solved by **sensitivity-based solution algorithm**, is setup for finding the most vulnerable location.

DEFINITIONS

Consider an **arbitrary-shaped region with multiple central business districts** (CBDs) as shown in Figure 1, in which the **road network is approximated as a continuum. Different classes of users**, who are **continuously distributed over the region**, will travel from their demand location to the CBDs along the least costly route within the two-dimensional space. Due to the differences of traveling environment in the surface road and expressway system, the expressways will be separately defined in the study region (Figure 1).

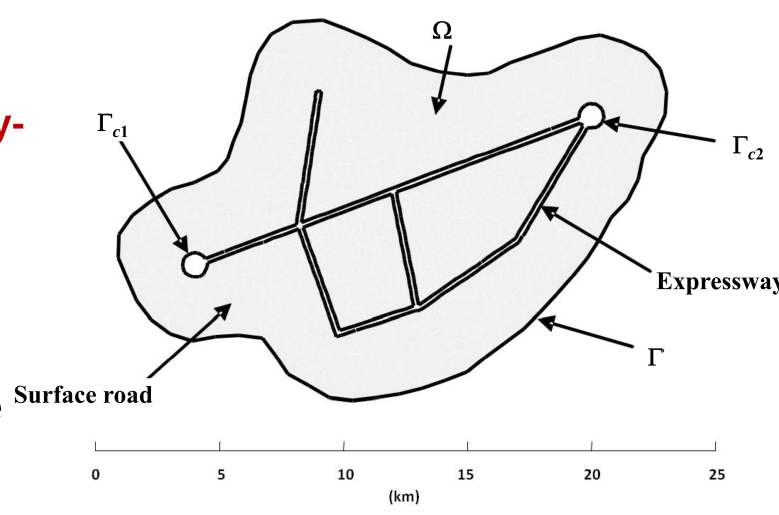

Figure 1 An example of the study region

Transportation cost function:

$$c_m(x,y)=a_m(x,y)+\frac{b_m(x,y)}{\upsilon(x,y)K(x,y)}\sum_i\sum_j\left|\mathbf{f}_{ij}(x,y)\right|$$

where $a_m(x,y)$ and $b_m(x,y)$ are respectively the free-flow and congestion-related parameter of class m users; $K(x,y)$ is the road density (in km/km²) of the non-degraded system; $\upsilon(x,y)$ is the percentage of road density remains after network degradation(s); $\mathbf{f}_{md}(x,y)$ is the flow vector of class m users heading to CBD d.

Percentage of road density remains under network degradation

$$\upsilon(x,y)=\prod_{j\in J}\upsilon_j(x,y),\quad \forall(x,y)\in\Omega$$

$$\upsilon_j(x,y)=\begin{cases}\frac{1}{r_j}\left(\bar{\upsilon}_j^{\gamma}-\upsilon_j^{\gamma}\right)\sqrt{\left(x_{jc}-x\right)^2+\left(y_{jc}-y\right)^2}+\upsilon_j^{\gamma}, & \forall(x,y)\in\tilde{\Omega}_j\\ 1, & otherwise\end{cases}$$

where $\upsilon_j(x,y)$ is the percentage of road density remains under network degradation j; $\tilde{\Omega}_j$ is the circular impact area of network degradation j; x_{jc} and y_{jc} are respectively the x- and y-coordinate for the center of network degradation j; r_j is the radius of the impact area for network degradation j; $\bar{\upsilon}_j^{\gamma}$ and υ_j^{γ} are respectively the percentage of capacity remains at the boundary and center of the impact area. In this study, it is **assumed that the degree of network degradation is highest**, or the remaining road density is lowest, **at the center of degradation** (i.e. $\upsilon_j^{\gamma}\le\bar{\upsilon}_j^{\gamma}$).

Accessibility index

$$AI=\frac{\sum_{m\in M}\sum_{d\in D}\iint_{\Omega}\frac{q_{md}(x,y)}{u_{md}(x,y)}d\Omega}{\sum_{m\in M}\sum_{d\in D}\iint_{\Omega}q_{md}(x,y)d\Omega}$$

where $u_{md}(x,y)$ and $q_{md}(x,y)$ are respectively the total travel cost (in HKD) and demand (in veh/hr/km²) of class m users traveling to CBD d from location. Base on this definition, **the most vulnerable location**, which has the largest increase in total travel cost, **is the location with minimum accessibility index as it degrades**.

REGIONAL VULNERABILITY ANALYSIS

Regional vulnerability analysis is formulated as a **bi-level model**.

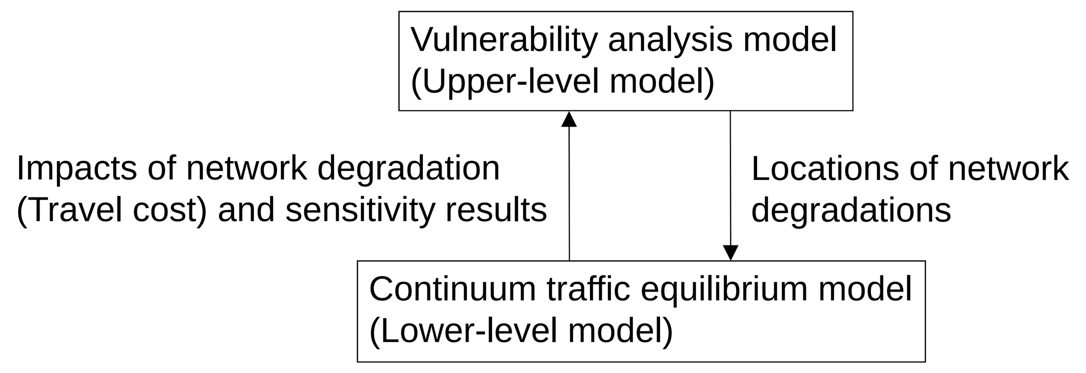

Figure 2 Bi-level model for regional vulnerability analysis

Lower level model

$$\left(a_m+\frac{b_m}{\upsilon K}\left(\sum_{i\in M}\sum_{j\in D}\left|\mathbf{f}_{ij}\right|\right)\right)\frac{\mathbf{f}_{md}}{\left|\mathbf{f}_{md}\right|}+\nabla u_{md}=0 \quad \text{User equilibrium condition}$$

$$\nabla\mathbf{f}_{md}+q_{md}=0 \quad \text{Flow conservation}$$

$$\mathbf{f}_{md}=\mathbf{0} \quad \text{Boundary condition for flow}$$

$$u_{md}=0 \quad \text{Boundary condition for cost}$$

Finite element method (FEM) and **Galerkin formulation** are respectively used to approximate the continuous variables and differential equations in the study region. With the approximations, **Newtonian algorithm** with **step size determination** is used to solve the equilibrium flow pattern and travel cost.

Upper level model

$$\underset{\Phi}{\text{Minimize}}\quad AI(\Phi)=\frac{\sum_{m\in M}\sum_{d\in D}\iint_{\Omega}\frac{q_{md}(x,y)}{u_{md}^{*}(x,y)}d\Omega}{\sum_{m\in M}\sum_{d\in D}\iint_{\Omega}q_{md}(x,y)d\Omega}$$

$$\text{Subject to}\quad (x_{jc},y_{jc})\in\Omega,\ \forall j\in J$$

where Φ is the vector of the coordinates for the center of degradations; u_{md}^{*} is the equilibrium travel costs from the lower-level model. The upper level model is **discretized** through the application of FEM and is solved by **convex combination method**. **Decent direction** of the above model is evaluated based on the results from the **sensitivity analysis of the lower level model**.

NUMERICAL EXAMPLES

Characteristics of continuum vulnerability analysis

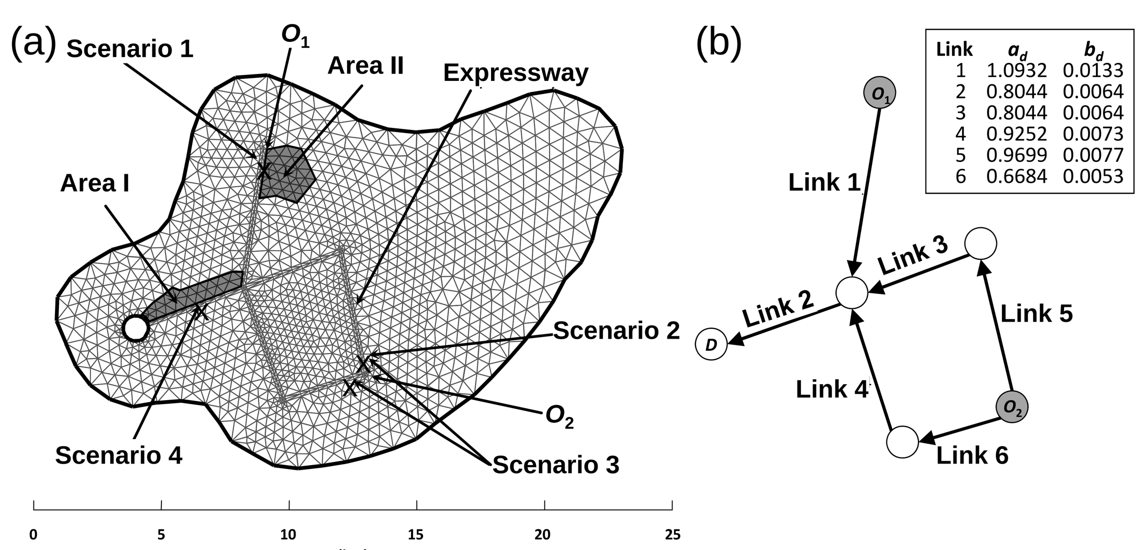

Figure 3 (a) The continuum network; (b) The discrete network

- 2 OD pairs, $(O_1,D)=3000$ veh/hr and $(O_2,D)=4000$ veh/hr
- **4 degradation scenarios** are considered
- **Base case** ($c_m^{Surface\ Road}=100\times c_m^{Expressway}$): Vulnerability analysis from the continuum and discrete network gives the **same ranking of the scenarios** (Based on the accessibility index). This **base case** is tried to **mimic the discrete network** and is used as a base for the comparisons in the 3 tests.

Impact of surface road network (Test 1)

- **Transportation cost of Area I** is assumed to be **twice** (instead of 100 times) of that of the expressway

Test 1	Average travel cost		Accessibility index	Ranking	Ranking (Base case)
	O_1	O_2			
No degradation	86.4	73.7	0.01333	-----	-----
Scenario 1	162.5	71.9	0.01072	2	3
Scenario 2	84. 7	94.7	0.01171	4	4
Scenario 3	85. 3	161.3	0.00908	1	2
Scenario 4	98.0	86.3	0.01138	3	1

(a) (b)

(a) Reduced by 50%~55% as compared to the base case. ∵Travelers detoured to Area I in this scenario (Figure 4)

(b) Average travel cost ↓ ⇒ Accessibility index ↑ ⇒ Lower rank (less vulnerable)

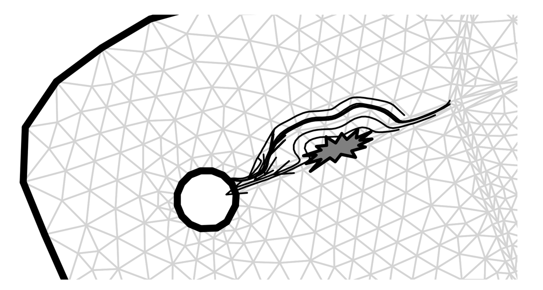

Figure 4 Detours in Area I

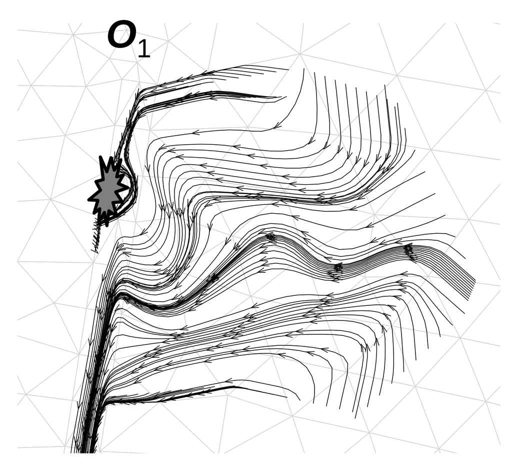

Figure 5 Paths in Area II

Impact of demand distribution (Test 2)

- **Demand from O_1 is evenly distributed on Area II**

Test 2	Average travel cost		Accessibility index	Ranking	Ranking (Base case)
	O_1	O_2			
No degradation	68.5	73.7	0.01413	-----	-----
Scenario 1	68.8	73.7	0.01410	4	3
Scenario 2	68.4	97.1	0.01224	3	4
Scenario 3	67.4	162.4	0.00989	2	2
Scenario 4	181.5	188.1	0.00540	1	1

(c) (d)

(c) Very minor increase as compared to no degradation. ∵Not all demand in Area II is affected by the degradation (Figure 5)

(d) Similar to **(b)**

Impact of the extent of degradation (Test 3)

- Degradations of **0.8 km, 1.6 km, 2.4 km and 3.2 km on link 2** (Scenario 4) are considered.
- In **discrete case**, such degradations will have **same impact** (i.e. link 2 is degraded)

Test 3		Test 1		
	Accessibility index		Accessibility index	Ranking
No degradation	0.01335	No degradation	0.01333	-----
0.8 km	0.01219	Scenario 1	0.01072	2
1.6 km	0.01106	Scenario 2	0.01171	4
2.4 km	0.01033	Scenario 3	0.00908	1
3.2 km	0.00984	Scenario 4	0.01138	3

(e) (f)

(e) Accessibility index decreases as longer detour on the surface roads

(f) Link 2 (Scenario 4) is becoming more vulnerable than link 1 (Scenario 1) if a degradation of over 2.4 km is considered

Finding the most vulnerable location

- $c_m^{Surface\ Road}=2\times c_m^{Expressway}$
- Demand is **continuously distributed in around O_1 and O_2**
- **Circular degradation** (500m radius) is considered
- Bi-level model is solved for most vulnerable location

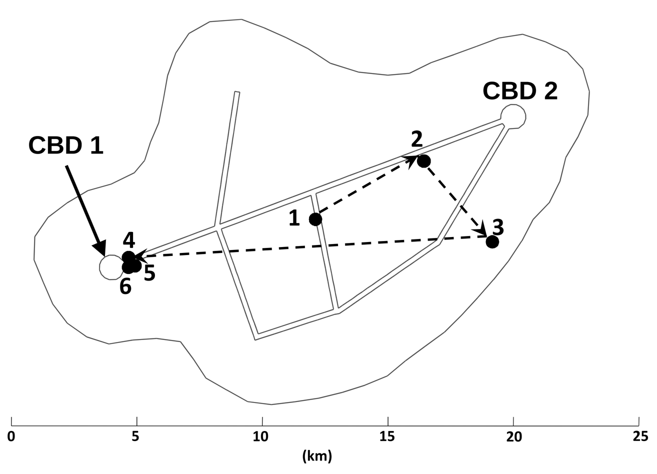

Figure 6 Search for the most vulnerable location

- Upper level model **converge in 5 steps** and takes **10.5 hr**
- **Most vulnerable location** is next to CBD 1 (**location 6**) despite the higher demand at CBD 2. ∵500m radius degradation **could not cause substantial** detour to the surface road near CBD 2

CONCLUSIONS

- Discrete modeling framework have problems of **alternative route availability**, **demand definition** and **degradation representation** in vulnerability analysis
- **Continuum model**, which could address these problems, could lead to a **different conclusion in vulnerability analysis**.
- Future research will be focused on developing **efficient algorithm for solving the bi-level optimization problem**

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