Traffic Flow Theory

Mijn presentatie spreekt over de verkeersstroom theorie !

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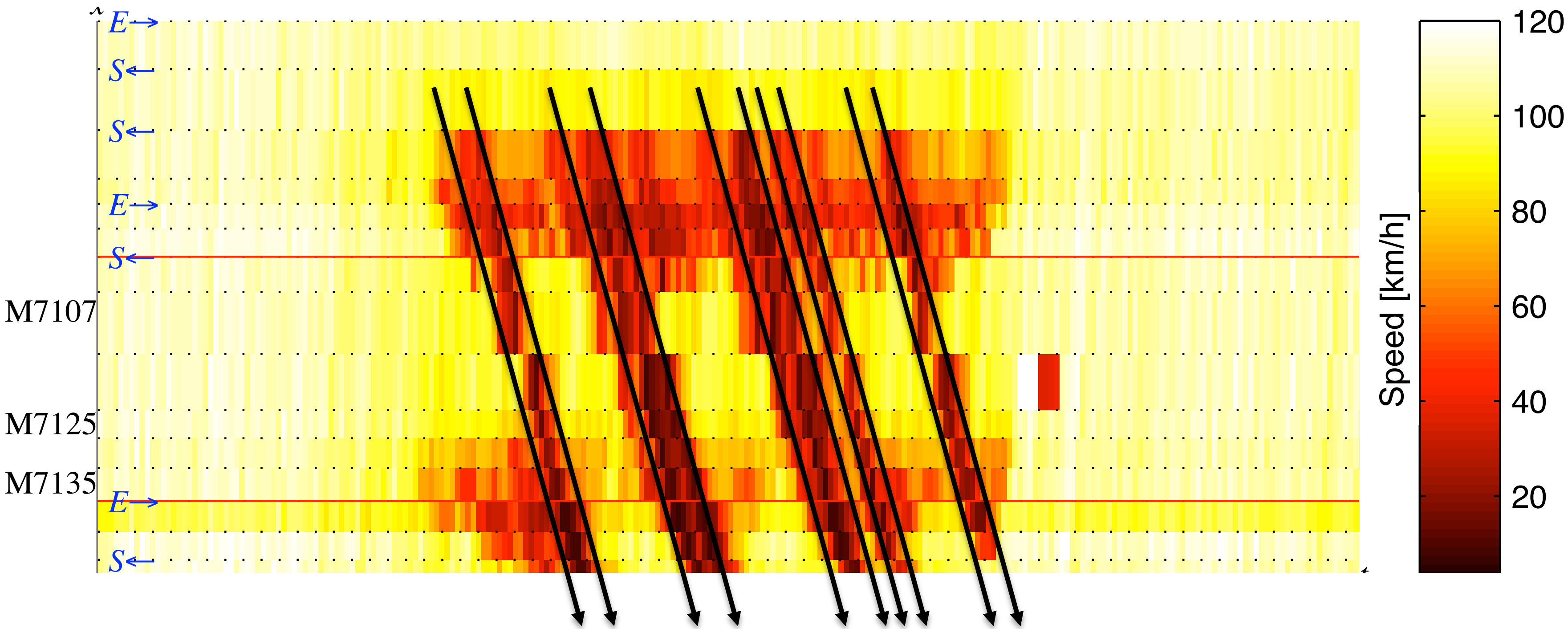
Outline

- Experimental evidences
  - Traffic behavior on freeways
  - The fundamental diagram
- Traffic modeling
  - The three representation of traffic flow
  - The three kinds of traffic models
  - Equilibrium (first order) model
- Overview of first order model solutions
- The variational theory
  - General basis
  - Connections between the three traffic representations
- Some extensions to the theory

Experimental evidences

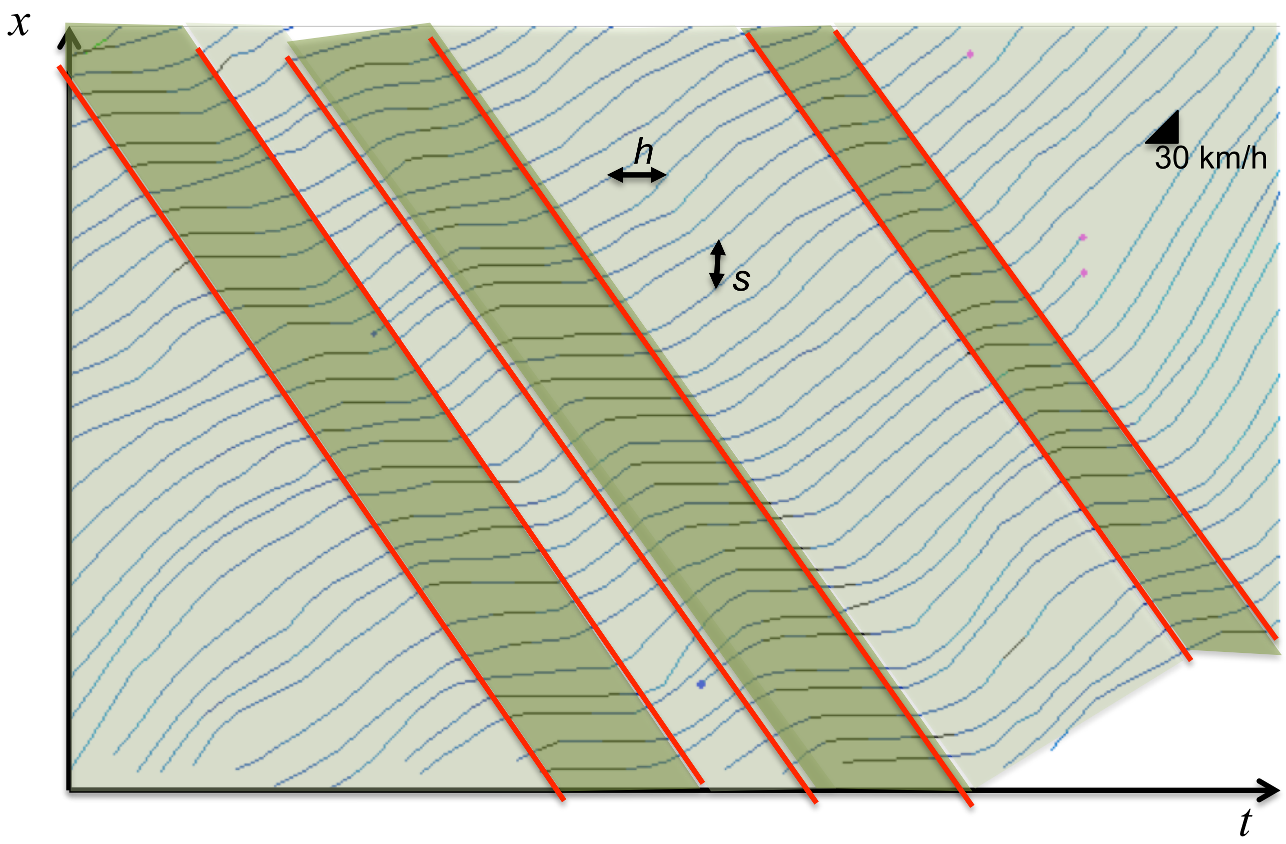
Traffic flow on a motorways (M6 in England)



Data were kindly provided by the Highway Agency

Traffic representation

NGSIM Data – I80/lane 4 – USA



Microscopic vision

Vehicle dynamics
position x
speed v

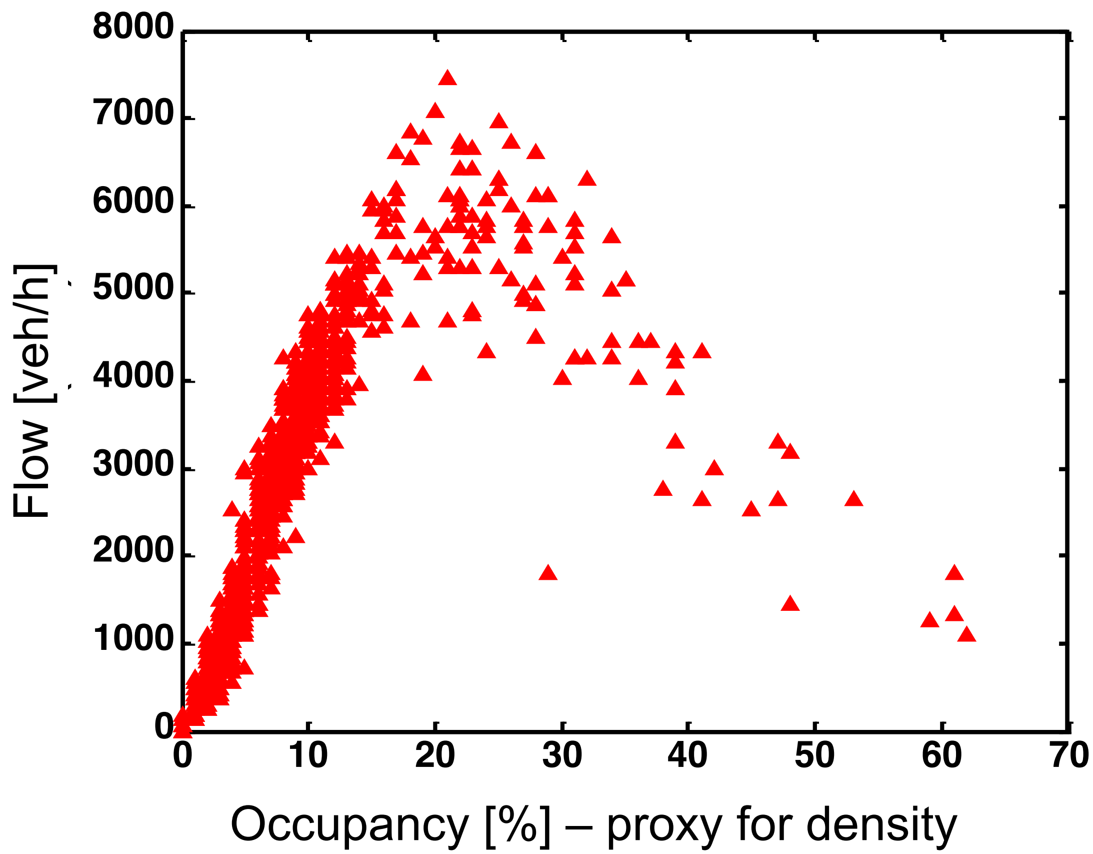
Interactions
spacing s
Headway h

Macroscopic vision

density k
flow q

Shockwaves

Flow / occupancy plot on a motorway (M6)



The fundamental diagram (FD)

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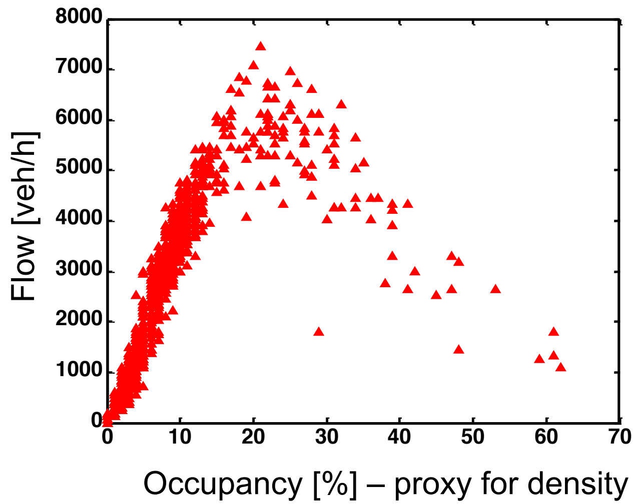
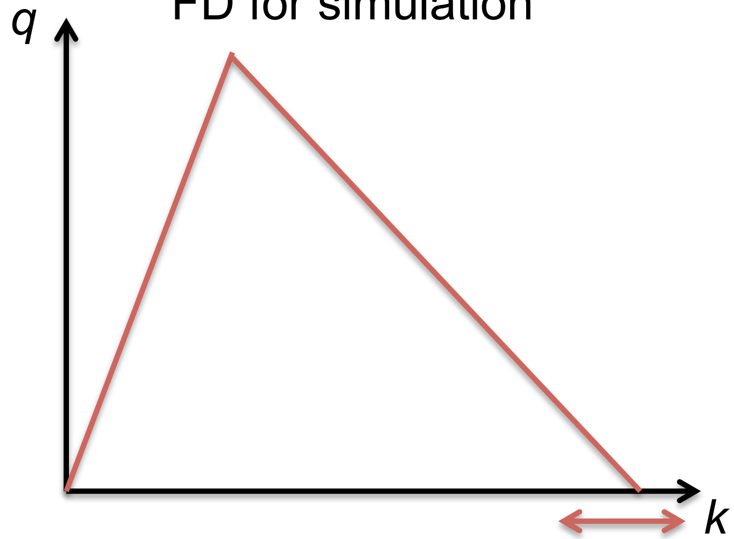
Different definitions of the FD

FD for monitoring



FD for simulation



Aggregation / impacts of local behavior (lane-changing, traffic composition,…)

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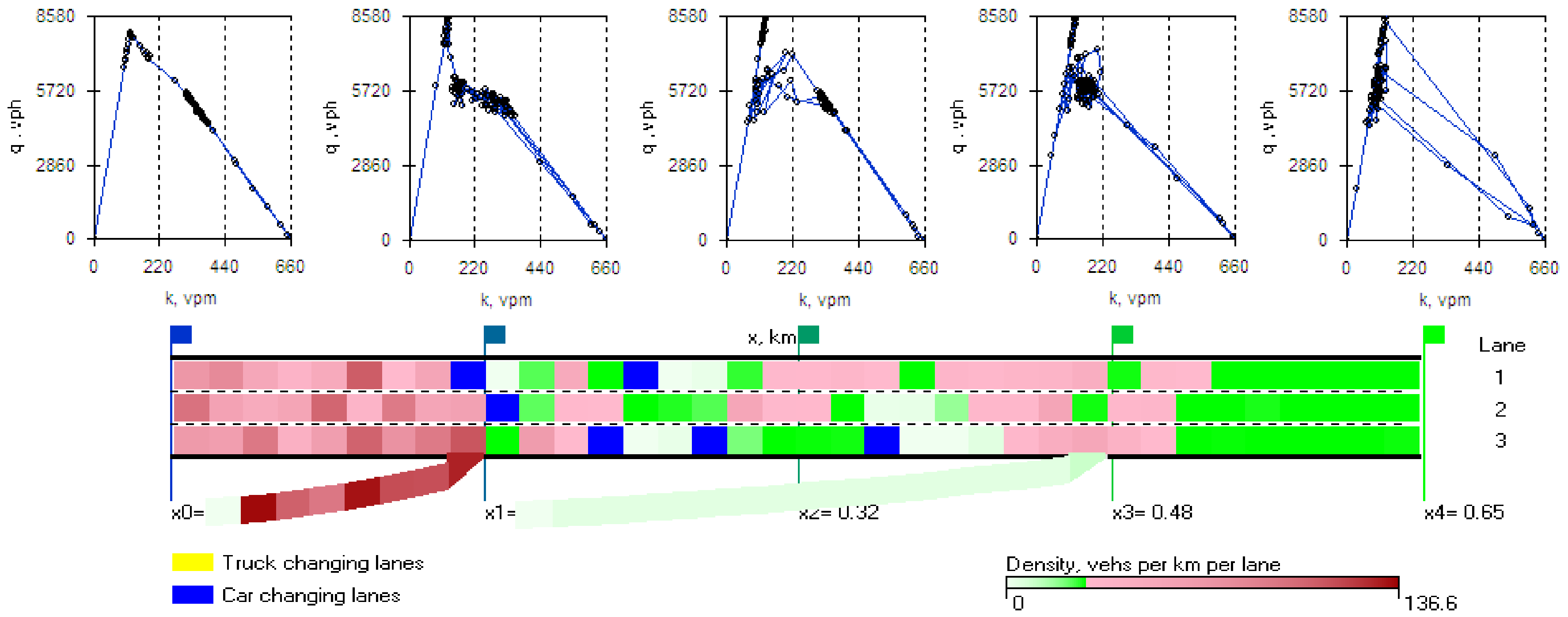
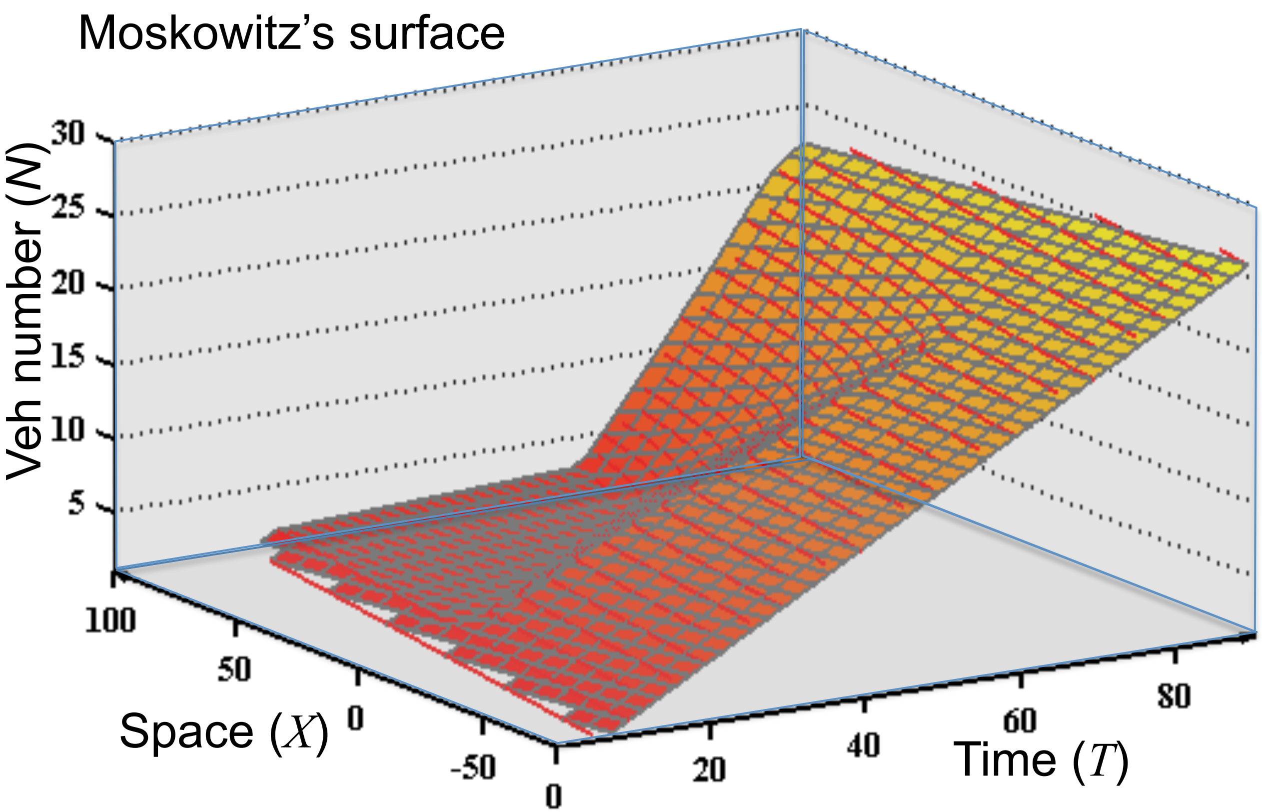
Impact of the lane agregation on the FD



Simulations are figures were kindly provided by Prof. Jorge Laval

Traffic Modelling

From discrete to continuous representations

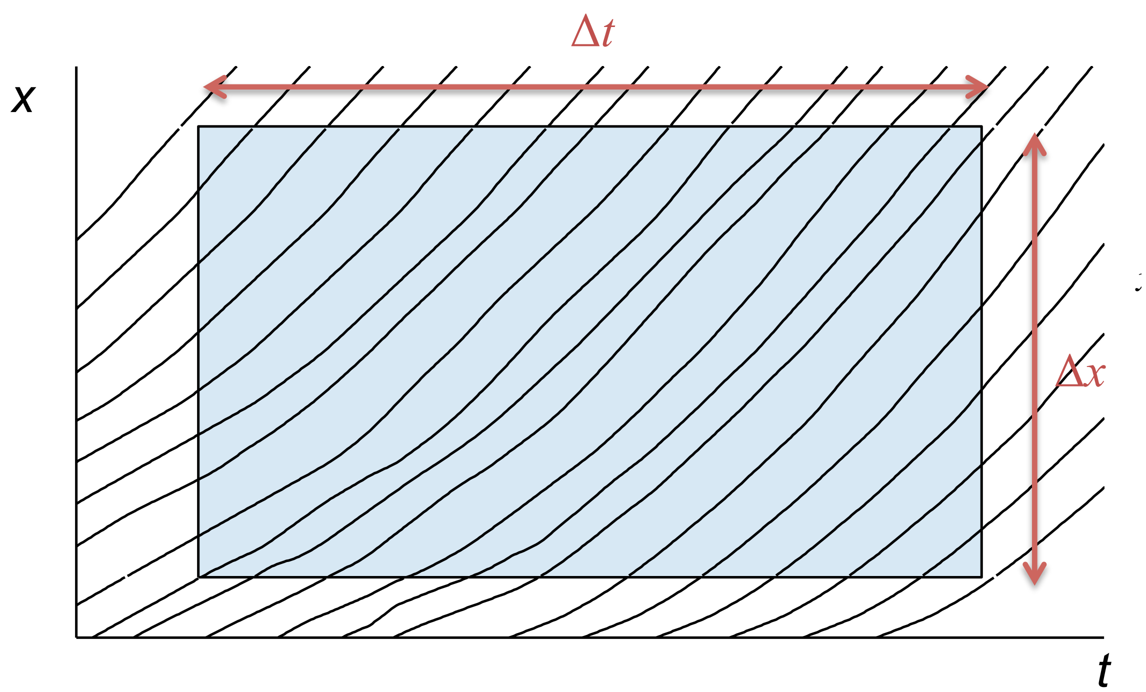


Eulerian coordinates
$N(t,x)$

Lagrangian coordinates
$X(t,n)$

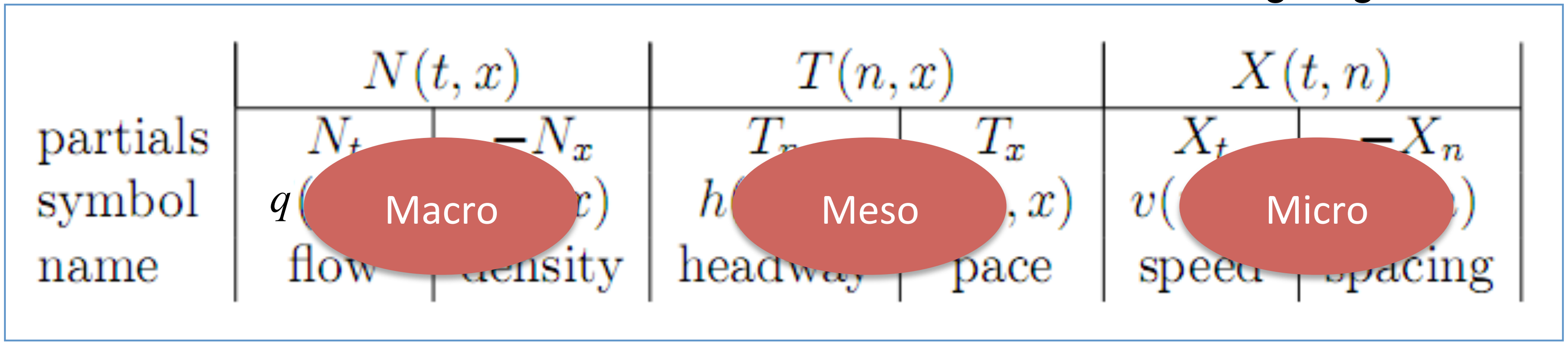
T coordinates
$T(t,n)$

From micro to macro: Edie's definitions



$$q_i = \sum_k d'_k \Big/ \Delta x \Delta t \quad \text{and} \quad k_i = \sum_k \tau'_k \Big/ \Delta x \Delta t$$

The three representation of traffic flow



	Eulerian		T coordinates		Lagrangian	
	$N(t,x)$		$T(n,x)$		$X(t,n)$	
partials	N_t	$-N_x$	T_n	T_x	X_t	$-X_n$
symbol	$q($[illegible]	[illegible]$)$	$h($[illegible]	[illegible]$, x)$	$v($[illegible]	[illegible]$)$
name	flow	density	headway	pace	speed	spacing

$N(t,x)$ # of vehicles that have crossed location x by time t

$X(t,n)$ position of vehicle n at time t

$T(n,x)$ time vehicle n crosses location x

(Laval and Leclercq, 2013, part B)

Classical classification of traffic models

- Macroscopic
  - Conti... ...tation
  - M...
  - ... (may be distinguished per class)
    ... order) / + transition states (2nd order)
- ... models
  - Discrete repr...
  - Mainly stocha...
  - Local interactions (car-following)
- Mesoscopic models
  - Discrete or semi-discrete representation
  - Intermediate level for traffic representation (vehicle clusters or link servers)

Duality

Eulerian coordinates

Lagrangian coordinates

T coordinates

Equilibirum model
(LWR)

For further details see the model tree
from (van Wageningen-Kessels, PhD, 2013)

Equilibrium macroscopic model (1)

- The PDE expression
  - in Eulerian coordinates

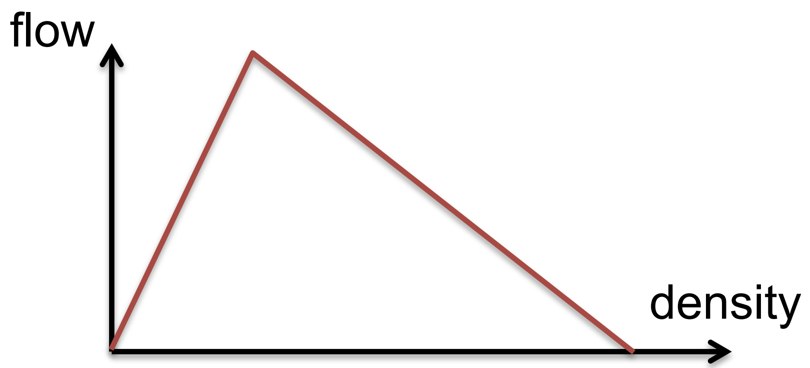
$$k_t + Q(k)_x = 0$$

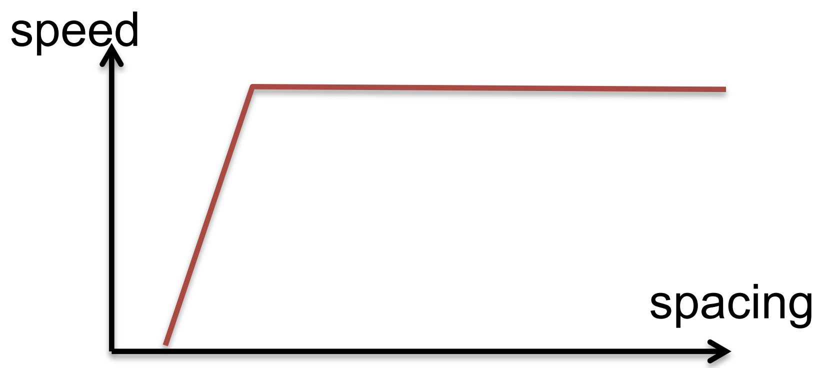
  - in Lagrangian coordinates

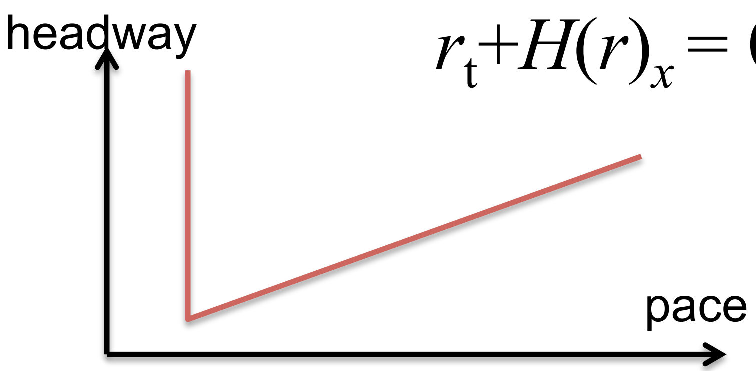
$$s_t + V(s)_x = 0$$

  - in T coordinates

$$r_t + H(r)_x = 0$$







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Equilibrium macroscopic model (2)

- The Hamilton-Jacobi (HJ) expression
  - In Eulerian coordinates

$$q=Q(k)$$

  - In Lagrangian coordinates

$$v=V(s)$$

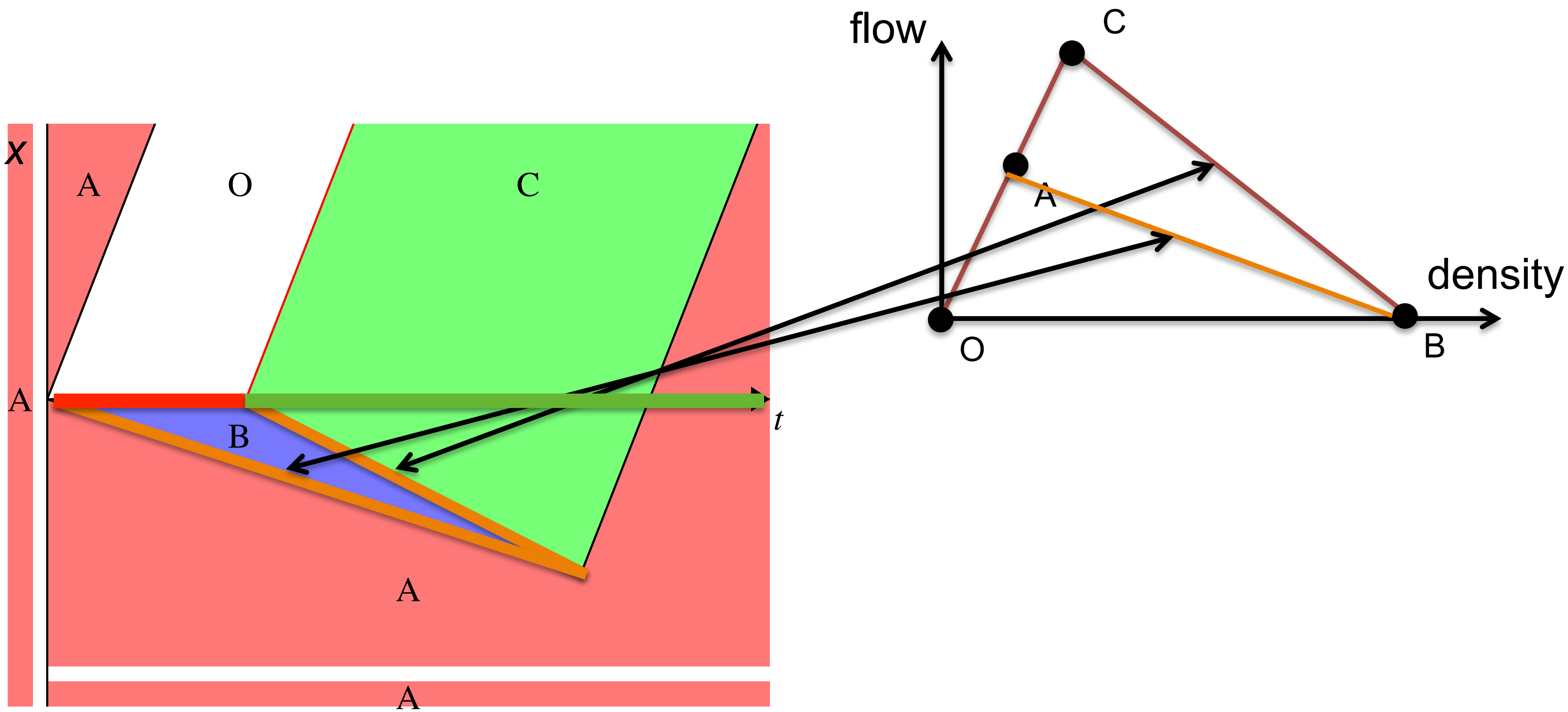
  - In T coordinates

$$h=H(r)$$

Appropriate expression of the FD

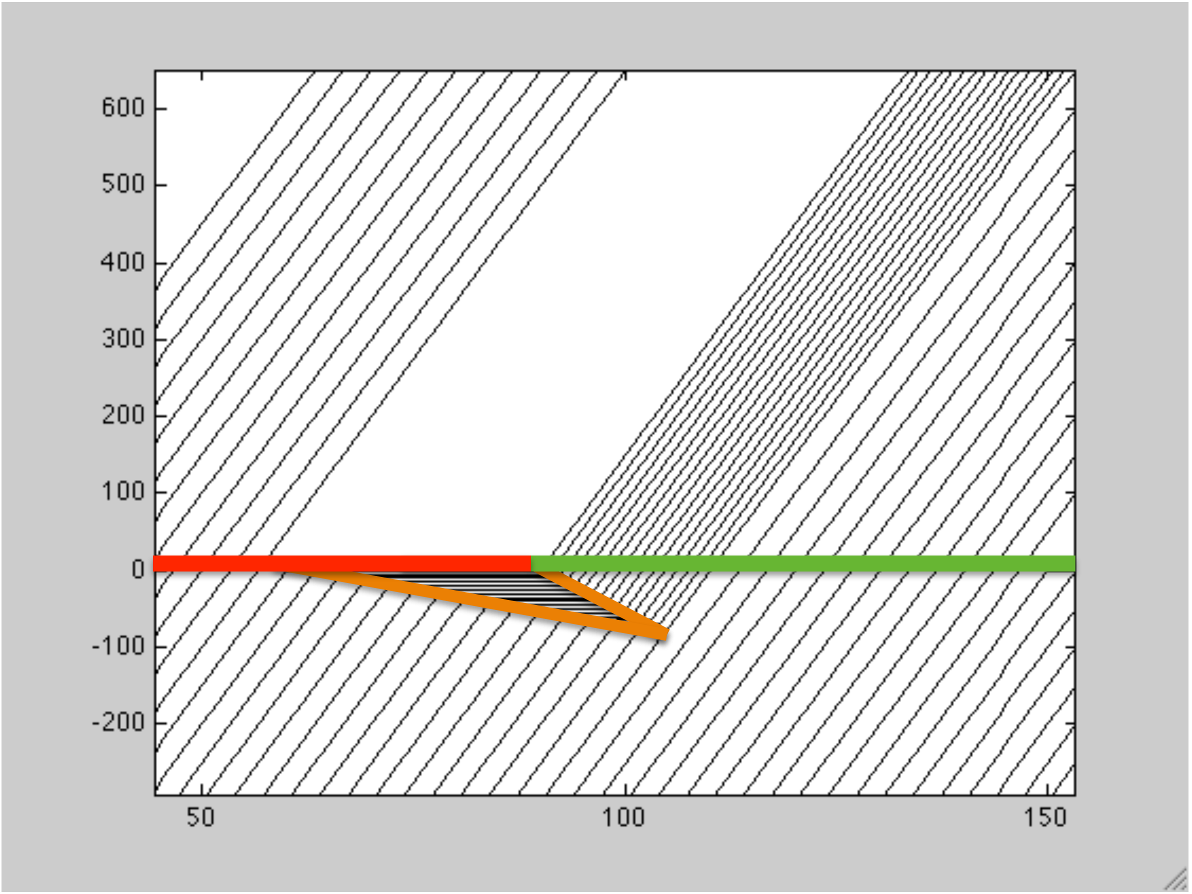
Overview of first order model solutions

Solutions for an unsaturated traffic signal (1)



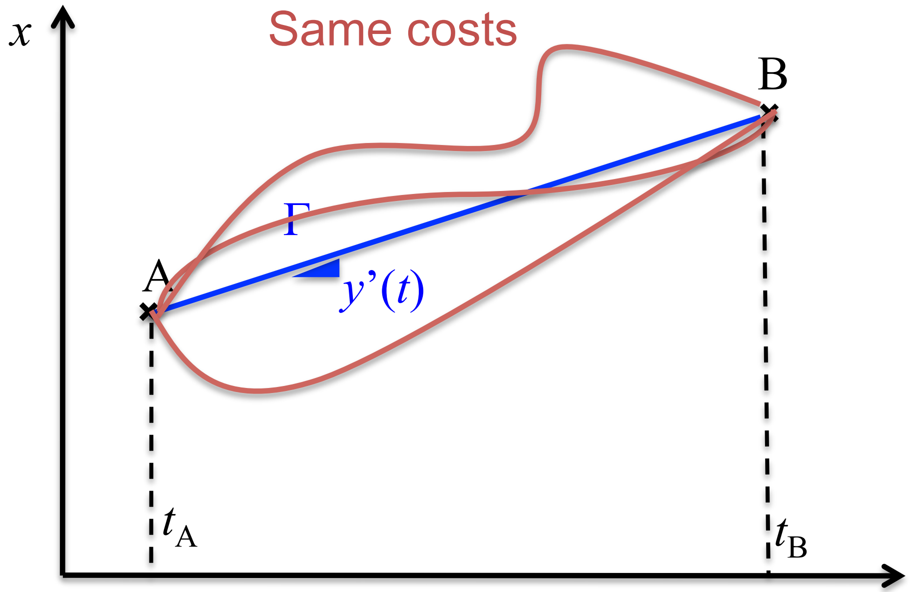
General solution methods:
Hyperbolic equations (EDP),
characteristics, waves,…

Solutions for an unsaturated traffic signal (2)



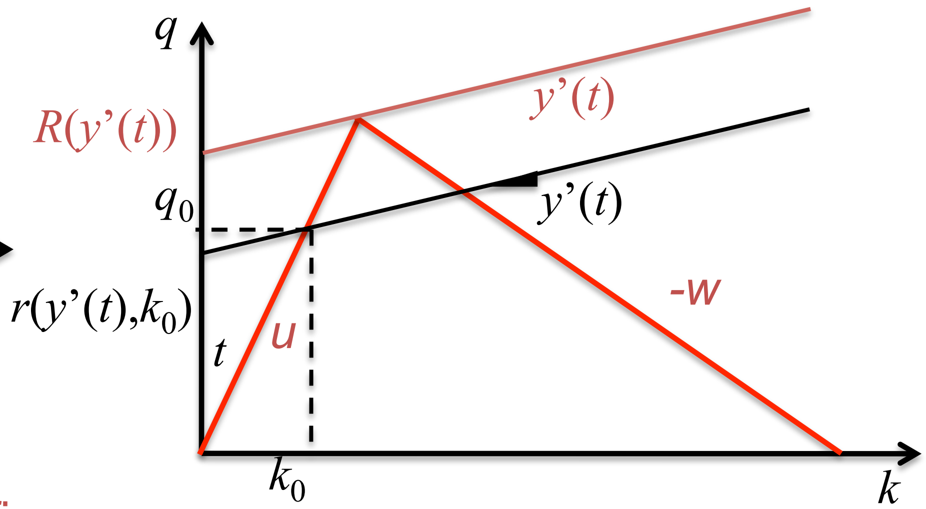
The Variational Theory

General considerations on the variations of *N*



$$\Delta N_{AB} = \int_{t_A}^{t_B} d_t N = \int_{t_A}^{t_B} \partial_t N + y'(t)\partial_n N$$

$$\Delta N_{AB} = \int_{t_A}^{t_B} \underbrace{Q(k(t)) - y'(t)k(t)}_{r(y'(t),k)}$$



Legendre's transformation:

$$r(y'(t),k) \leq R(y'(t)) = \sup_k \left(r(y'(t),k) \right)$$

This makes costs independent from traffic states but no longer from the paths

Equality is observed on the optimal wave paths

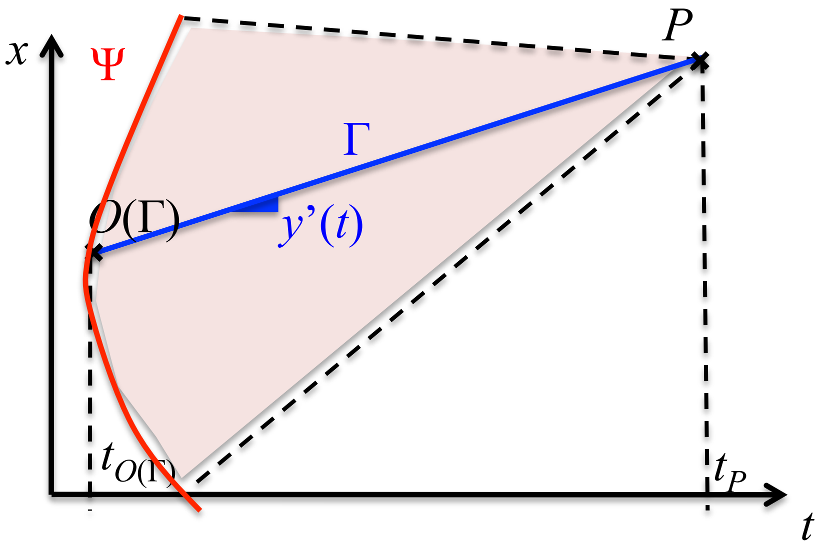
Variational theory (VT) in Eulerian – General basis

HJ Equation: $q = Q(k) \Leftrightarrow \partial_t k = Q(-\partial_x k)$

General expression for the solutions:

$$N_P = \min_{\Gamma \in D_P} \left(N_{O(\Gamma)} + \Delta(\Gamma) \right)$$

$$\Delta(\Gamma) = \int_{t_{O(\Gamma)}}^{t_P} r(y'(t), k)\, dt$$



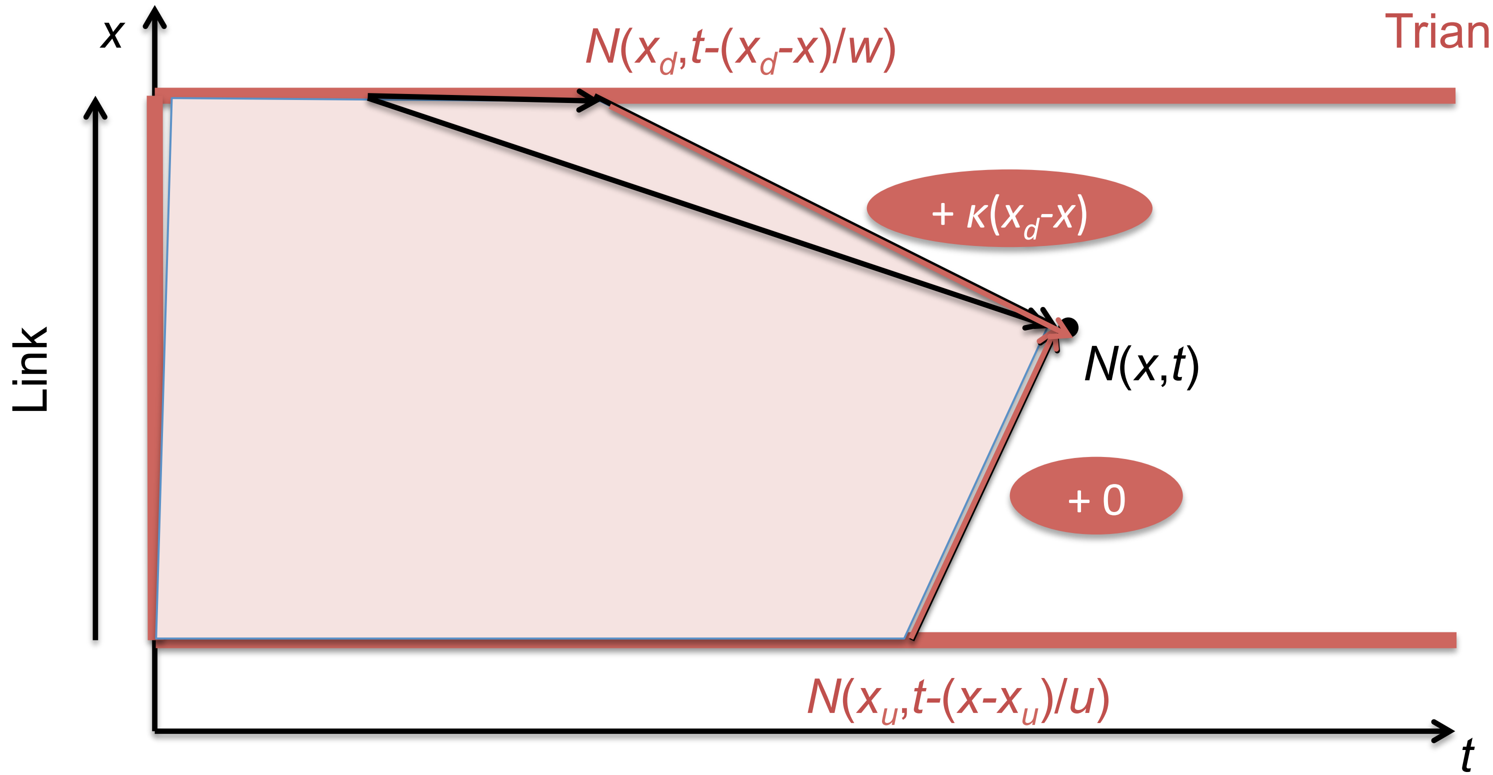
Key VT result using the Legendre's transformation

$$N_P = \min_{\Gamma \in D_P} \left(N_{O(\Gamma)} + \Delta'(\Gamma) \right)$$

$$\Delta'(\Gamma) = \int_{t_{O(\Gamma)}}^{t_P} R(y'(t))\, dt$$

VT is really useful with PWL FD (and especially triangular one)

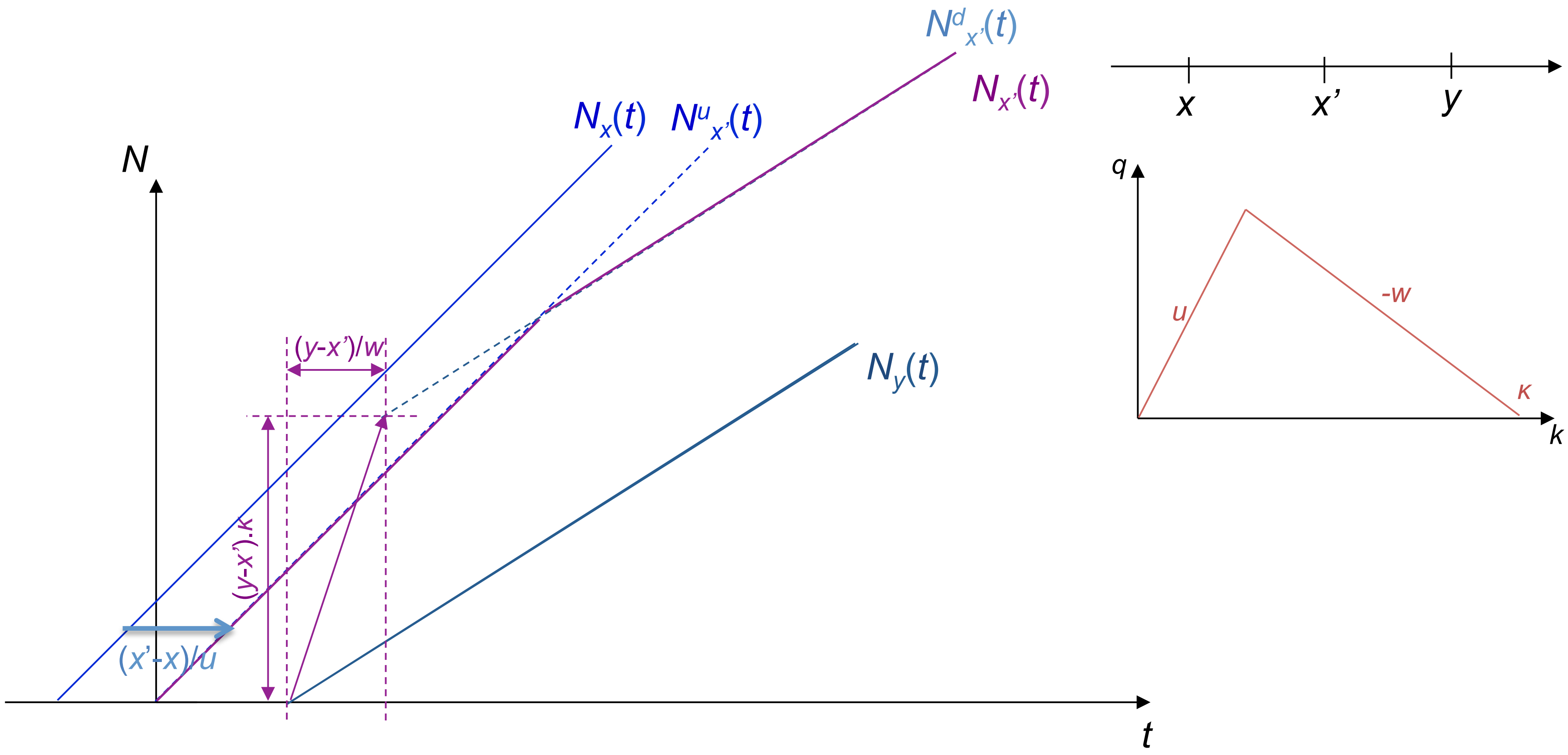
VT in Eulerian – The Highway Problem



$$N(x,t)=\min\left[\underbrace{N\left(x_u,t-\frac{(x-x_u)}{u}\right)}_{free-flow},\underbrace{N\left(x_d,t-\frac{(x_d-x)}{w}\right)+\kappa(x_d-x)}_{congestion}\right]$$

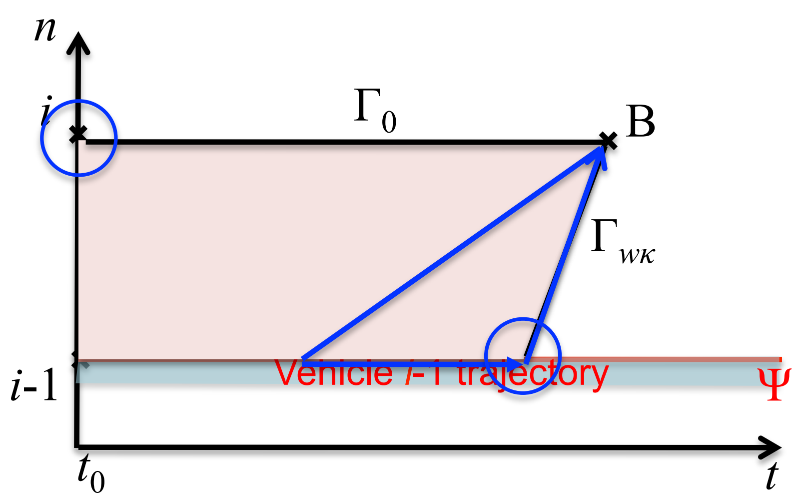
Newell's model (1993) !!!

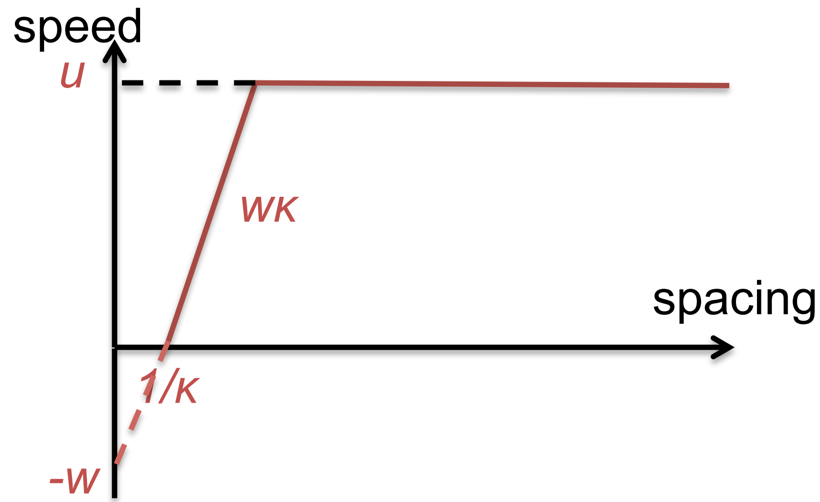
Classical formulation of the Newell's N-curve model



Well-known as the three dectectors problem

VT in Lagangian - the IVP problem



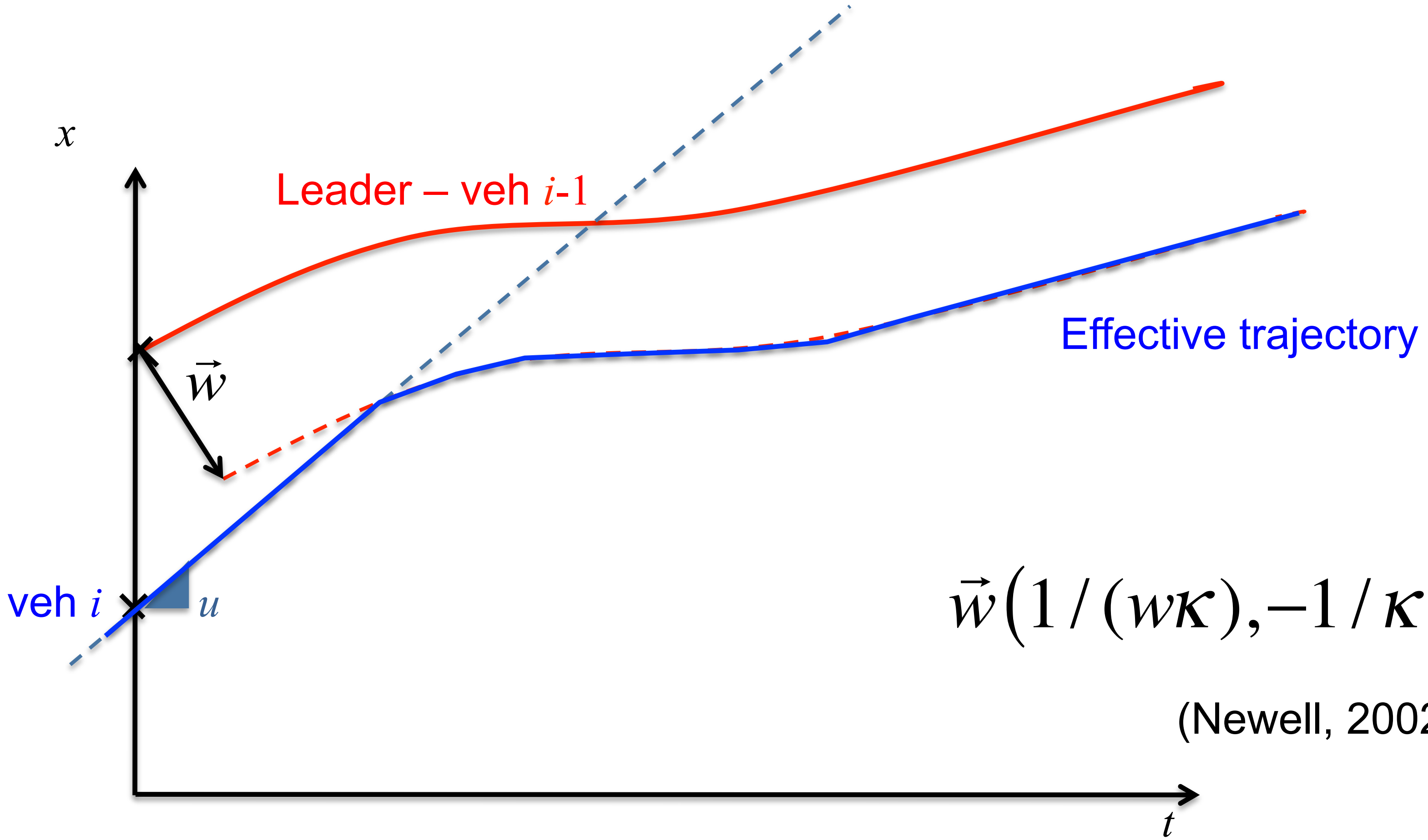


$$X_B = \min\left(X_{O(\Gamma_0)} + \Delta(\Gamma_0), X_{O(\Gamma_{w\kappa})} + \Delta(\Gamma_{w\kappa})\right)$$

$$X_B = X(t,i) = \min\left(\underbrace{X\left(t_0,i\right) + u.(t-t_0)}_{free-flow}, \underbrace{X\left(t-1/(w\kappa), i-1\right) - w.(1/w\kappa)}_{congestion}\right)$$

Newell's model again !!! (2002)

Classical formulation of Newell's car-following model

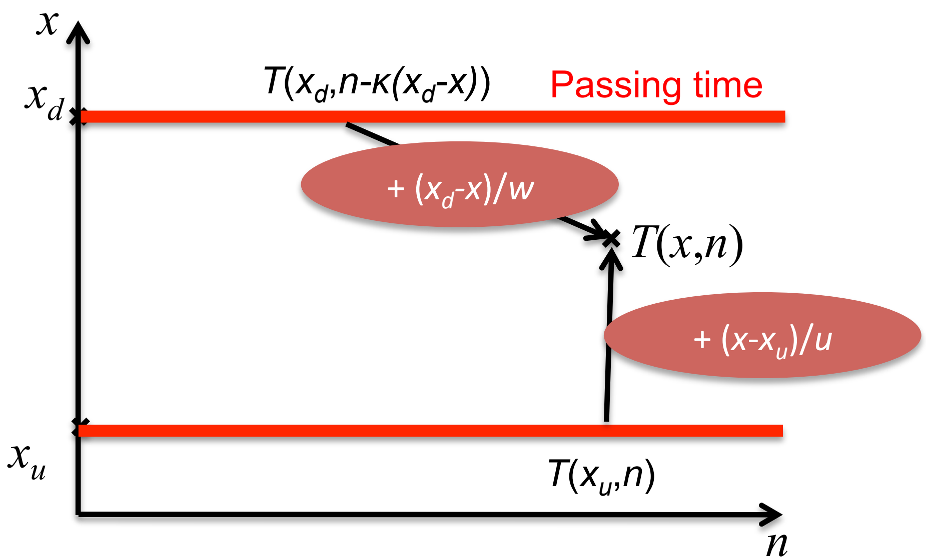


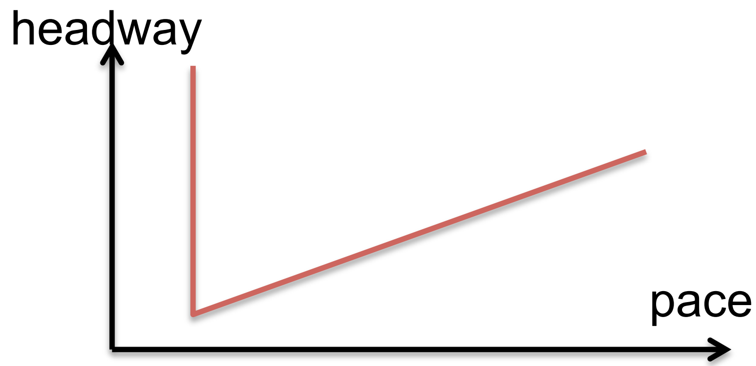
$$\vec{w}\left(1/(w\kappa),-1/\kappa\right)$$

(Newell, 2002)

The simplest car-following rule
Account for driver reaction time

VT in T coordinates

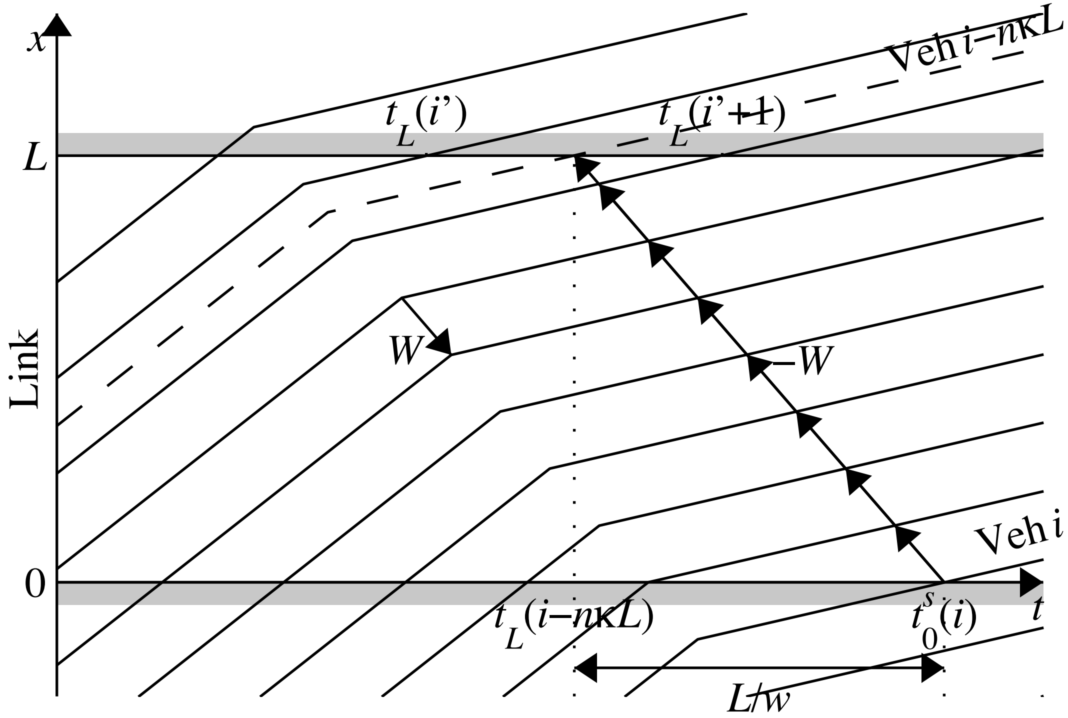




$$T(x,n) = \max\left(\underbrace{T(x_u,n) + {}^{(x-x_u)}\!/_{u}}_{free-flow}, \underbrace{T(x_d, n-\kappa(x_d - x)) - {}^{(x_d - x)}\!/_{w}}_{congestion} \right)$$

Mesoscopic model
(Mahut, 2000; Leclercq & Becarie, 2012)

The mesoscopic LWR model

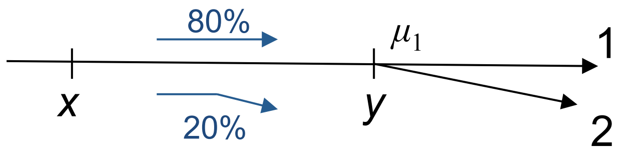


Variational theory - summary

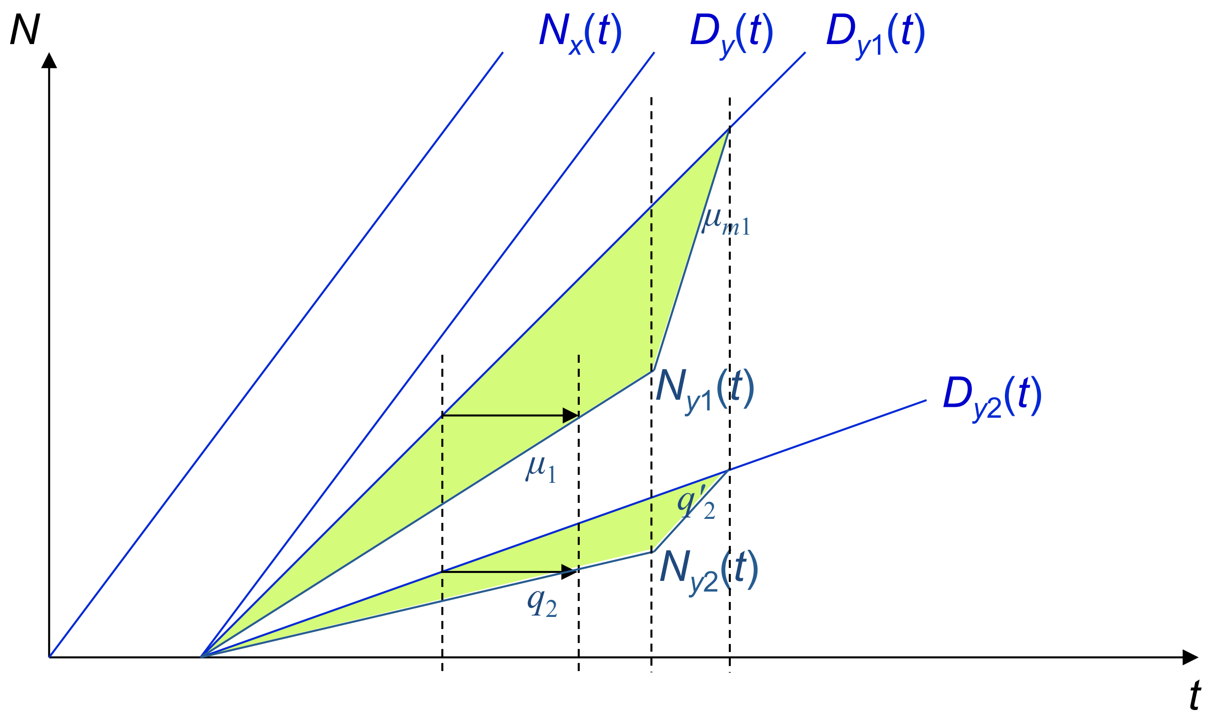
- Variational theory exhibits the connections between the three traffic representations for the LWR model
- A unique model that leads to three solution methods (numerical scheme) corresponding to the three different vision on traffic flow
  (macroscopic / mesoscopic / microscopic)
- Some previous models appears to be particular cases for the LWR model and a triangular fundamental diagram in different systems of coordinates

Extensions to the theory

Diverge: Newell's FIFO model

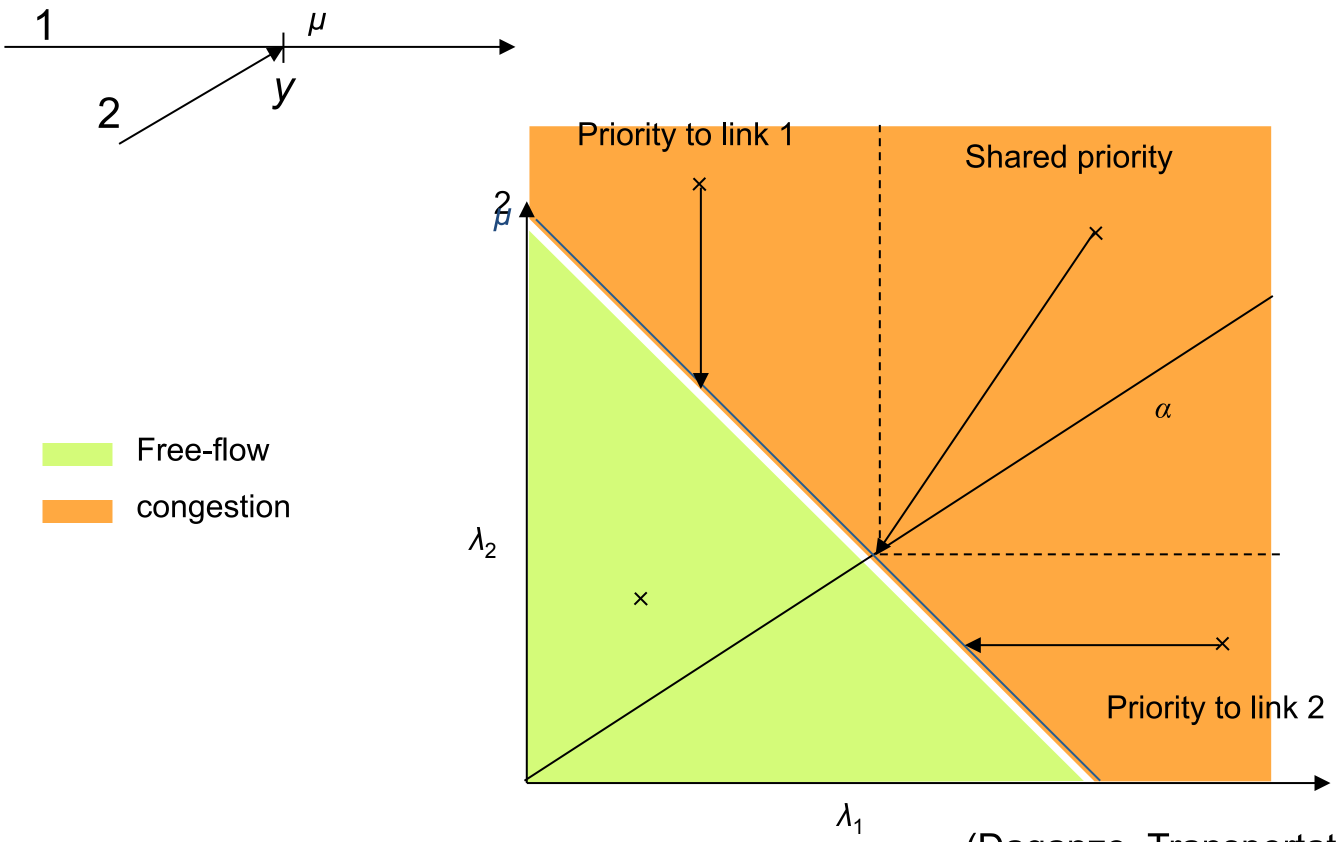


(Newell, 1993)



FIFO => Travel times should be equal whatever the destination is

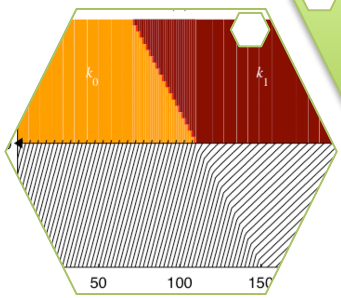
Merge: Daganzo's model



(Daganzo, Transportation Research part B, 1995)

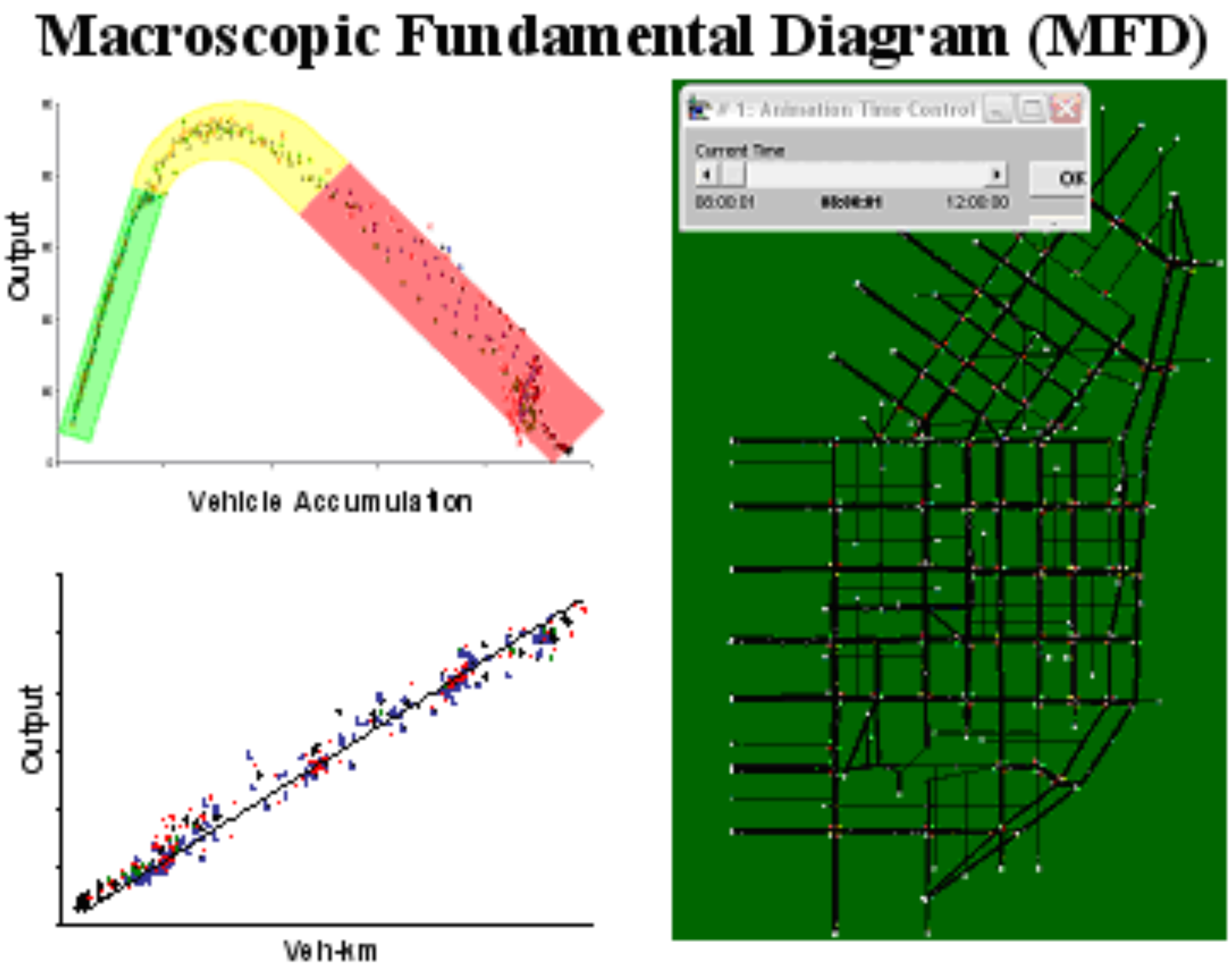
This model has been proved consistent with experimental observations a multitude of times

Other extensions



Need to be coupled with a route choice model to simulate a network

The network fundamental diagram



(Daganzo & Geroliminis, 2008)

Thank you for your attention

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References

- Daganzo, C.F., 2006b. In traffic flow, cellular automata = kinematic waves. *Transportation Research B*, **40**(5), 396-403.
- Daganzo, C.F., 2005. A variational formulation of kinematic waves: basic theory and complex boundary conditions. *Transportation Research B*, **39**(2), 187-196.
- Daganzo, C.F., 2005b. A variational formulation of kinematic waves: Solution methods. *Transportation Research B*, **39**(10), 934-950.
- Daganzo, C.F., 1995. The cell transmission model, part II: network traffic. *Transportation Research B*, **29**(2), 79-93.
- Daganzo, C.F., 1994. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research B*, **28**(4), 269-287.
- Daganzo, C.F., Menendez, M., 2005. A variational formulation of kinematic waves: bottlenecks properties and examples. In: Mahmassani H.S. (Ed.), *16th ISTTT*, Elsevier, London, 345-364.
- Edie, L.C., 1963. Discussion of traffic stream measurements and definitions. In: J. Almond (Ed.), 2nd ISTTT, OECD, Paris, 139-154.
- Laval, J.A., Leclercq, L. The Hamilton-Jacobi partial differential equation and the three representations of traffic flow, *Transportation Research part B*, **2013**, accepted for publication.
- Leclercq, L., Laval, J.A., Chevallier, E., 2007. The Lagrangian coordinates and what it means for first order traffic flow models. In: Allsop, R.E., Bell, M.G.H., Heydecker, B.G. (Eds), *17th ISTTT*, Elsevier, London, 735-753.
- Gerlough, D.L. et Huber M.J. *Traffic flow theory – a monograph*. Special report n°165, Transportation Research Board, Washington. **1975**.
- Lighthill, M.J et Whitham, J.B. On kinematic waves II. A theory of traffic flow in long crowded roads. *Proceedings of the Royal Society*, **1955**, Vol A229, p. 317-345.
- Newell, G.F., 2002. A simplified car-following theory: a low-order model. *Transportation Research B*, **36**(3), 195-205.
- Newell, G.F., 1993. A simplified theory of kinematic waves in highway traffic, I general theory; II queuing at freeway; III multi-destination. *Transportation Research B,* **27**(4), 281-313.
- Richards, P.I., 1956. Shockwaves on the highway. *Operations Research*, **4**, 42-51.

Exercices

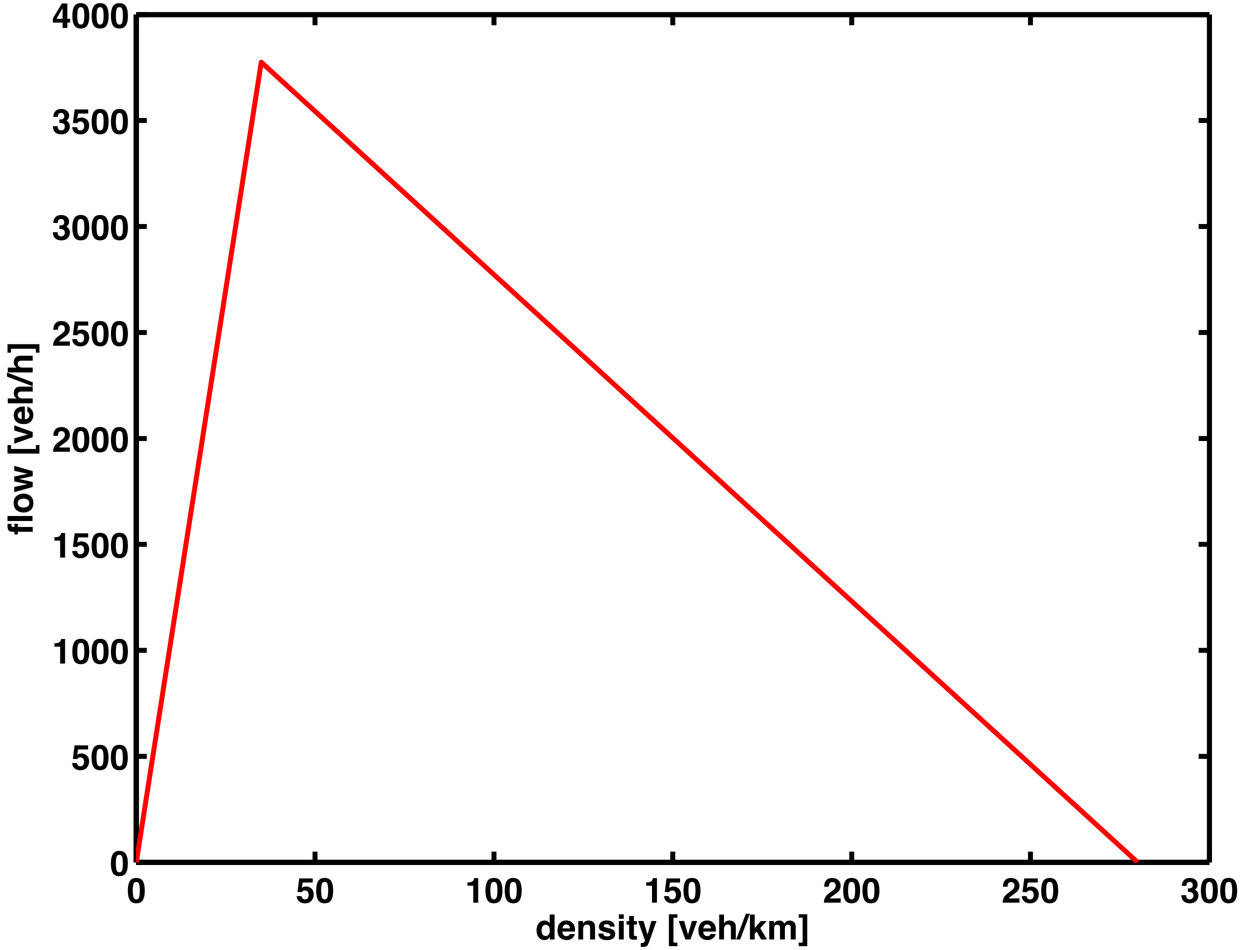
Problem statement

Let consider a freeway with two lanes and the following FD: u=30 m/s ; w=4.28 m/s ; κ=0.28 veh/m. Two points a et b are respectively located at x=0 m and x=3600 m.

The flow at *a* is constant and equal to 3000 veh/h. At time t=120 s, the capacity at is reduced from 1800 veh/h during 10 minutes.

- Draw the fundamental diagram
- Determine the *N*-curve at x=3600, x=1800, x=600 and x=0 m
- Provide an estimate for the maximal length of the congestion

The fundamental diagram



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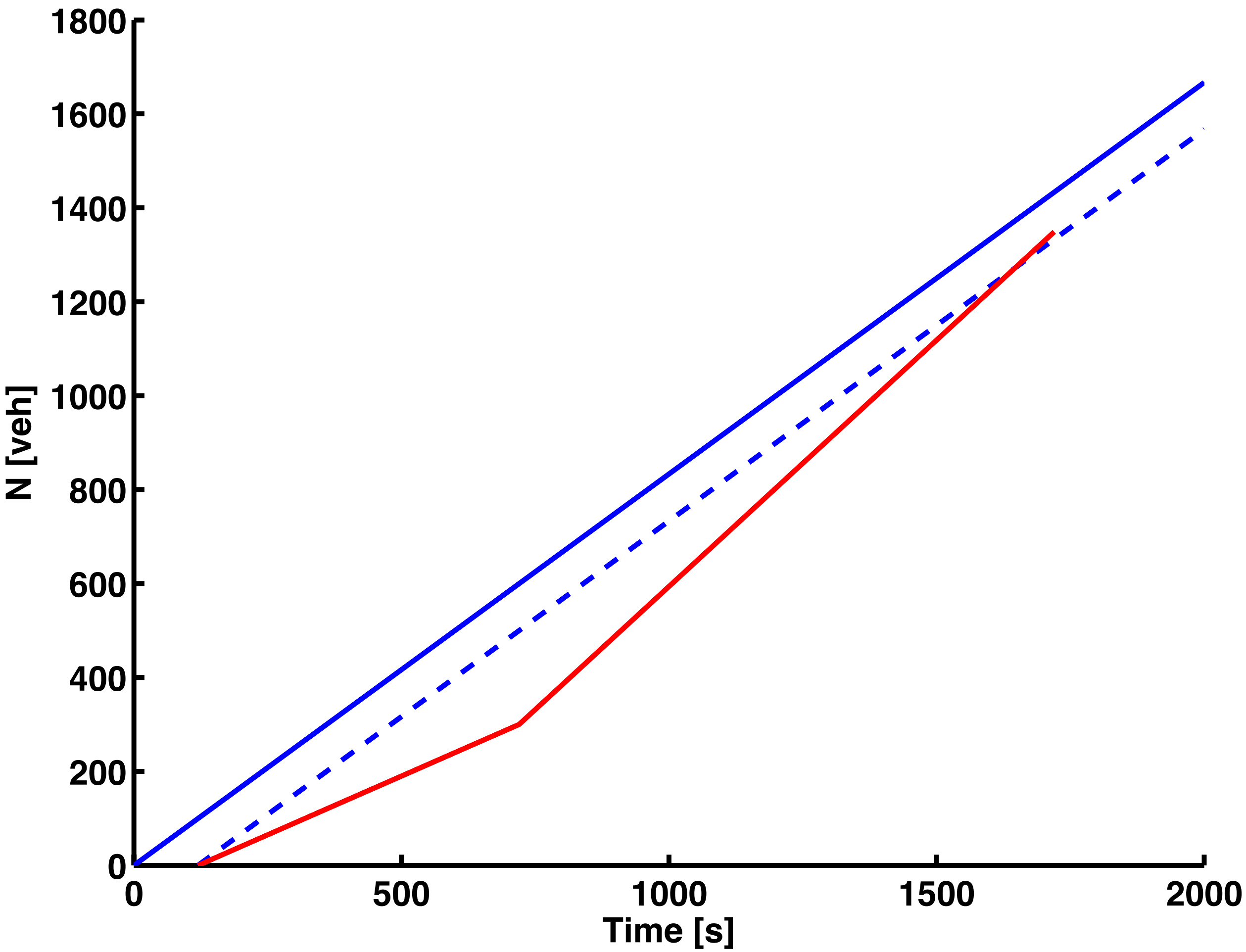
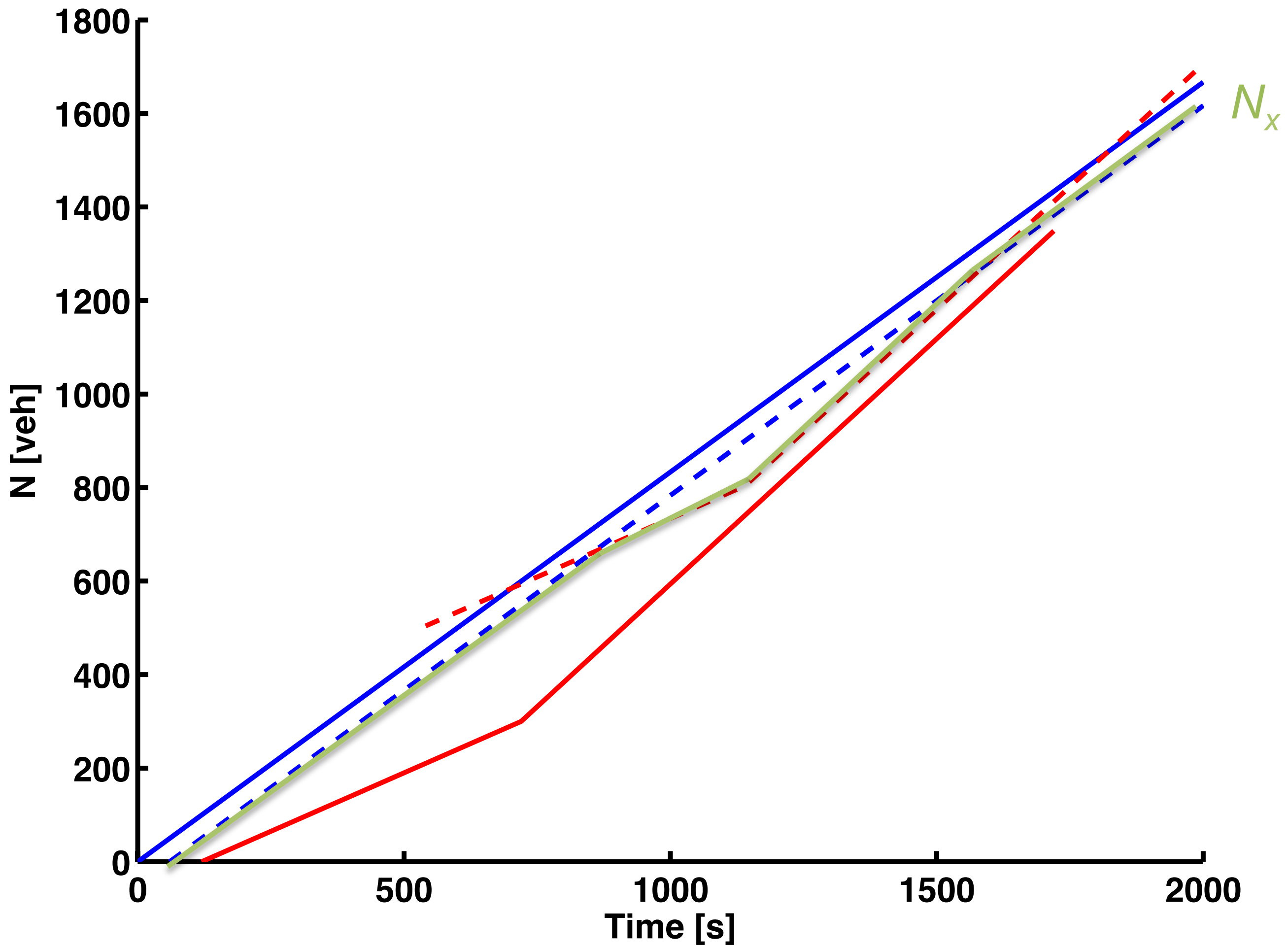
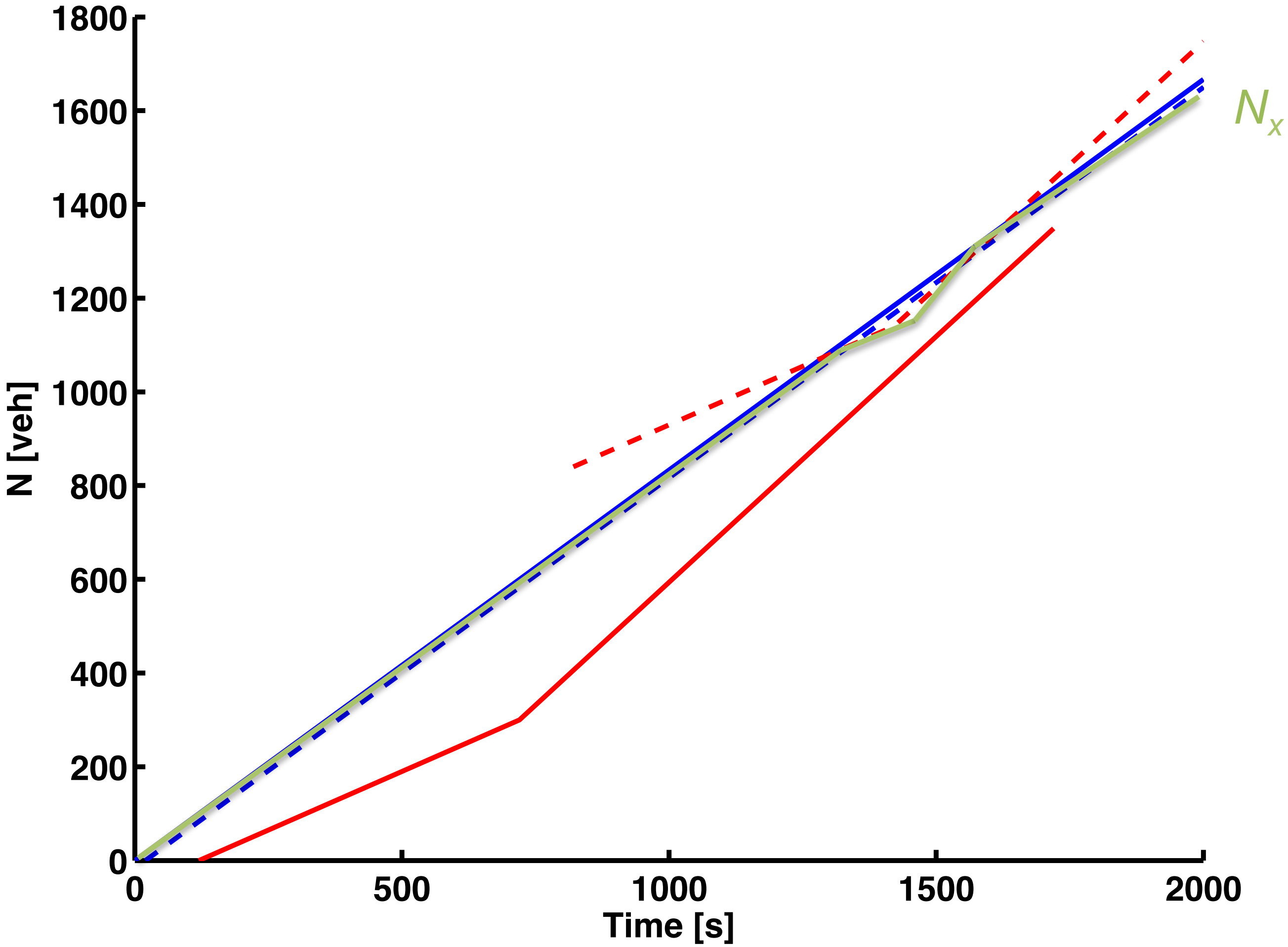
N-curve at *x*=3600 m



N-curve at *x*=1800 m



N-curve at *x*=600 m



N-curve at x=0 m