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Outline

1. Macroscopic Modeling of Urban Networks
2. Multi-region MFD-based Model
3. Dynamic Route Guidance
   - High-level Optimal Routing Scheme
   - Objective Function
   - Model Predictive Control Framework
4. Case Study
   - Set-up
   - Results
5. Concluding Remarks and Future Research
Modeling large-scale urban networks would be a complex task if one wants to study and model dynamics of every single element.

Control using such detailed modeling approach would be a tedious task.

⇒ Aggregate models
Urban network partitioned into multiple regions, each represented by an MFD
\( \mathcal{J}_i \): set of neighboring regions of region \( i \)

Flow from region \( i \) to region \( j \in \mathcal{J}_i \) is min. of 3 elements:

1. Demand from region \( i \) to region \( j \), \( D_{i,j} \)
2. Supply in region \( j \), \( S_j \)
3. Capacity of boundary between region \( i \) & region \( j \), \( C_{i,j} \)
Accumulation (veh/km/lane)
Production (veh/h/lane)
Macroscopic Fundamental Diagram
0 50 100 ...

and in some regions the average speed will be high while in
others we observe low speeds. The objective function

The flow can be separated per destination. So, similar to

Therefore, the accumulation in any region

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**Supply function**

\[ S_j(k) = \begin{cases} P_{j,\text{crit}} & \text{if } n_j(k) \leq n_{j,\text{crit}} \\ P_j(n_j(k)) & \text{if } n_j(k) > n_{j,\text{crit}} \end{cases} \]

\( P_j(n_j(k)) \): production determined from MFD

**Demand function**

\[ D_{i,j}(k) = \sum_{d \in D} \left( \alpha_{i,j,d}(k) \cdot \frac{n_{i,d}(k)}{n_i(k)} \cdot P_i(n_i(k)) \right) \]

\( D \): set of all destinations
Update equations

\[ n_{i,d}(k + 1) = n_{i,d}(k) + \frac{T_s}{\sum_{\lambda \in \Lambda_i} \kappa_{\lambda} L_{\lambda}} \left( \sum_{j \in \mathcal{J}_i} q_{j,i,d}(k) - \sum_{j \in \mathcal{J}_i} q_{i,j,d}(k) \right) \]

Total accumulation in region \( i \):

\[ n_i(k + 1) = \sum_{d \in \mathcal{D}} n_{i,d}(k + 1) \]
- Regional destinations
- Optimal splitting traffic towards neighboring regions
- Aims:
  - avoid congestion in intermediate regions
  - decrease the overall travel time
High-level Optimal Routing Scheme

Objective Function

Model Predictive Control Framework

Multi-region MFD-based Model

Dynamic Route Guidance

Case Study

Concluding Remarks and Future Research
Minimizing total travel delay:

\[ J_{TD} = T_s \cdot \sum_{i \in R} \sum_{k=0}^{K-1} \left( \sum_{\lambda \in \Lambda_i} \kappa_{\lambda} L_{\lambda} \right) \cdot n_i(k) \]
Optimal Dynamic Route Guidance: A Model Predictive Approach Using MFD

Local controllers/
Urban regions

\( \alpha_{i,j,d}(k_c) \)

Optimization

Prediction

J

OD Table

\( n_{i,d}(k) \)

\( \alpha_{i,j,d}(k_c) \)

(multi-region) model
\[
J_{\text{TD}}^\text{MPC} = T_s \cdot \sum_{i=1}^{R} \sum_{k=M \cdot k_c}^{M \cdot (k_c+N_p)-1} \left( \left( \sum_{\lambda \in \Lambda_i} \kappa_\lambda L_\lambda \right) \cdot n_i(k) \right)
\]

– Overall optimization problem:

\[
\min \alpha_{i,j,d}(k_c) J_{\text{TD}}^\text{MPC}
\]

subject to:

model equations,

\[0 \leq \alpha_{i,j,d}(k) \leq 1,\]

\[\alpha_{i,j,d}(k) = \alpha_{i,j,d}^c(k_c), \quad \text{if } k \in \{M \cdot k_c, \ldots, M \cdot (k_c + 1) - 1\}\]
Blue squares: origins
Red circles: destinations
For each region, the MFD is approximated by:

\[ P_i = n_i \cdot V_{\text{free}} \cdot \exp \left( -\frac{1}{2} \left( \frac{n_i}{n_{\text{crit}}} \right)^2 \right) \]

**Table**: Origin-destination demands* (veh/h)

<table>
<thead>
<tr>
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<th>Region 2</th>
<th>Region 8</th>
<th>Region 9</th>
<th>Region 14</th>
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</tr>
<tr>
<td>Region 16</td>
<td>2000</td>
<td>1000</td>
<td>1000</td>
<td>1800</td>
</tr>
</tbody>
</table>

*: noise corrupted in the simulation model (network)
Determining splitting rates

- Static shortest-path (in time), Floyd-Warshall algorithm based on average speed of regions
- Shortest-path algorithm, updated every 60 seconds
- Dynamic, MPC algorithm using multi-region MFD model
Results for 4x4 network: (a) Uncontrolled (fixed routes), (b) Shortest-path algorithm, (c) Optimal dynamic routing using MPC
High-level scheme for optimal dynamic route guidance using MFD-based multi-region model

Optimal splitting rates towards neighboring regions

Avoiding detailed modeling and hence decreasing computational complexity of route guidance

Lower level control should be properly designed & connected to the high-level scheme

Multi-level scheme needs to be validated using real networks’ layouts and empirical data