Estimating MFDs in Simple Networks with Route Choice

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Outline

• Short recap of existing estimation methods for the Macroscopic Fundamental Diagram (MFD)
• Defining an advanced variational method for estimating MFD on a single hyperlink
• Testing the influence of route choices on the MFD for a parallel network
  – Static case
  – Dynamic case
• Conclusion
MFD definition

FD + Network structure (topology / signal timings) + Route choices = MFD
Existing estimation methods

**Experimental methods**
- Edie’s definitions from trajectories
- Aggregation of local information from loop detectors
- Mean speed information gained from probe vehicles

**Analytical methods**
- Variational Theory (VT) and practical cuts (Daganzo and Geroliminis, 2008) (Geroliminis and Boyacı, 2012)

Only provide a tight upper bound when the network is homogenous and regular
Advanced variational method for estimating MFD on single hyperlink
Estimating MFD on a hyperlink

- **Definitions**
  - Hyperlink: series of $m$ successive links ended by traffic signals
  - Homogeneous traffic conditions, i.e. no or well-balanced turning flow
  - FD parameters: free-flow speed $u$, wave speed $w$, jam density $\kappa$

\[ \text{link } i \quad \text{hyperlink} \]

\[ \left\{ \frac{G_i}{C_i}, \delta_i \right\} \]
Analytical estimation of MFD

Moving observer with mean speed $V$
Maximum passing rate $R(V)$

Cut: $Q = \min_V (KV + R(V))$

(Daganzo and Geroliminis, 2008)
Defining Time Windows

We only consider a discrete set of moving observers with speed \{V_k\}

One travel across the hyperlink at \( V_k \) defines the time window \( T_j(V_k) \)

Travelling across infinite copies of the hyperlink is equivalent to travelling across successive time windows: \( R(V_k) = \text{mean}(R_j(V_k)) \)

One only has to calculate \( R_j(V_k) \) over a sufficient number of \( T_j \) (law of large numbers)
Calculating $R_j(V_k)$ with VT

To minimize costs, the observer has to maximize the time spent on red phases – Shortcuts theory (Daganzo and Menendez, 2005)

Internal subpaths with $v < u$ may be replaced at same costs.

A graph can simply be constructed to explore all the possible paths
Sufficient variational graph \((V_k>0)\)

The graph is defined by three types of edges:

- **edge (a):** red phase (cost 0)
- **edge (b):** green phase (cost \(s=u\omega k/(u+w)\))
- **edge (c):** path with speed \(u\) that starts at the end of red phases (cost 0)

This graph is proved to be sufficient.

A classical shortest path algorithm provides \(R_j(V_k)\).

The same graph can be used for all \(V_k>0\) (only the ending points change).
Sufficient variational graph \((V_k<0)\)

A similar sufficient graph can be constructed when \(V_k<0\).

Edges (c) have a slope \(-w\) and a cost \(w\kappa\) in that case.
Resulting MFD for an example

\[ u = 20 \text{ m/s} \]
\[ w = 5 \text{ m/s} \]
\[ \kappa = 0.2 \text{ veh/m} \]

Hyperlink (\( m = 5 \))

\[ G_i = 75 \text{ s} \]
\[ C_i = 100 \text{ s} \]
\[ \delta_i = i \times 62 \text{ s} \]

Stationary cut

MFD
Influence of route choices on the MFD
Estimating MFD on a parallel network

- Different route choice assumptions
  - User Equilibrium (UE) (Wardrop, Logit)
  - System Optimum (SO)
- Different traffic conditions
  - Static equilibrium state (free-flow and congestion)
  - Dynamic loading (trapezoidal time dependent demand)
UE and SO calculations

• Static conditions (constant upstream demand or downstream supply)
  – UE: we first derive $v_i = F_i(v_1)$ and then calculate the eMFD
    \[
    \begin{align*}
    q &= \sum_i q_i = \sum_i Q_i(v_i) = \sum_i Q_i(F_i(v_i)) \\
    n &= \sum_i n_i = \sum_i \frac{q_i L_i}{v_i} = \sum_i \frac{Q_i(F_i(v_i)) L_i}{F_i(v_i)}
    \end{align*}
    \]
  – SO: equilibrium is defined by
    \[
    \begin{align*}
    \min \left( \sum_i q_i \tau_i \right) &= \min \left( \sum_i n_i \right) \\
    \sum_i q_i &= q \text{ with } q_i \geq 0 \text{ and } q_i = Q_i(n_i)
    \end{align*}
    \]

• Dynamic conditions
  – System dynamics is described by:
    \[
    \frac{dn_i}{dt} = f_i - Q_i(n_i)
    \]
  – We then apply the equilibrium rule

Details on aggregation methods and proofs are provided in the paper.
Static User Equilibrium ($M=2$)

MFDs

Flow distribution

$\frac{n_1}{n}$

$\frac{n_2}{n}$

free-flow
capacity
congestion

 accumulation [veh] 0 50 100 150 200 250 300

flow [veh/s] 0 0.2 0.4 0.6 0.8

$eMFD - \text{Logit (}\theta=0.1)$

$eMFD - \text{Wardrop}$

$\text{MFD1}$

$\text{MFD2}$

$\text{Wardrop}$

$\text{Logit (}\theta=0.1)$
Static System Optimum ($M=2$)

Flow distribution

Points with multiple solutions

MFDs
Static UE and SO ($M=3$)
Dynamic loading ($M=3$)

Split ratios differently evolve during the onset and the offset of congestion.

Final network state depend on the initial condition.
Conclusion

• Contrary to previous statements, route choices influence the MFD especially close to the network capacity

• UE and SO are close in congestion => few potential savings with traffic management

• Static SO is not always reachable through dynamic loading

• Further researches are needed to extend results to more complex networks and to account for heterogeneous traffic conditions
Thank you for your attention

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Some remarks

• The variational graphs include the practical cuts (observers that stop every $k$ signals) defined in (Daganzo and Geroliminis, 2008)

• The variational graphs provide a tight bound for the MFD without any regularity conditions

• The method is still restricted to homogeneous case. Otherwise, it only provide an upper bound.